Vrije Universiteit Brussel

Bruface
Brussels Faculty of Engineering

ULB
UNIVERSITÉ LIBRE DE BRUXELLES

MASTER THESIS

Micro-mechanical characterization of the capillary effective stress in granular materials

Roos Evenepoel

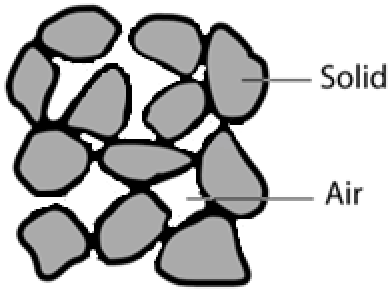
INTRODUCTION

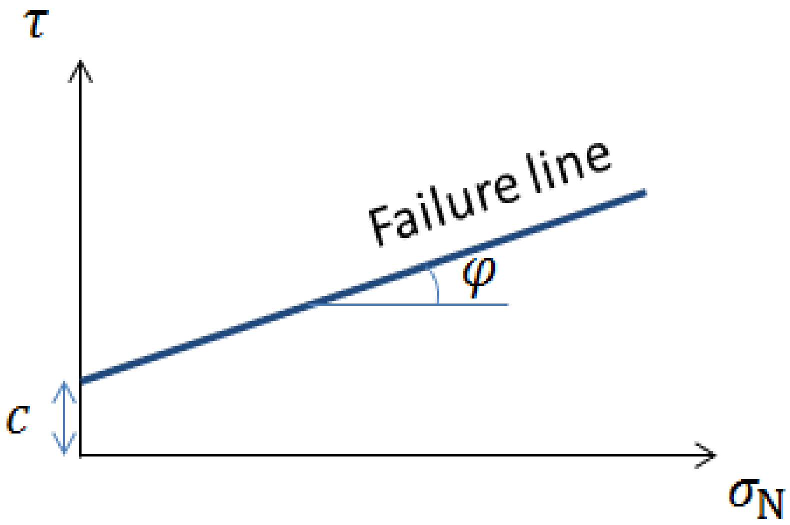
State of the art

Shear strength

- Dry soil

$$\tau_{\text{max}} = c + \sigma_{\text{N}}.\tan\varphi$$





- Unsaturated soil

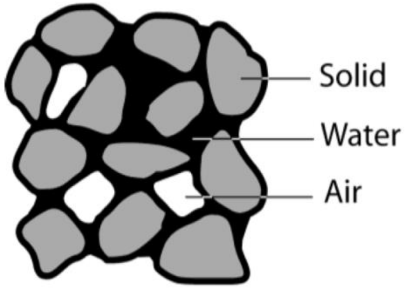
$$\tau_{\text{max}} = c + \sigma_{\text{N}}'.\tan\varphi$$

Bishop:

$$\sigma_{\text{N}}' = \sigma_{\text{N}} + \chi\Delta u$$

Until now:

$$\boxed{\chi = S_{\text{R}}}$$



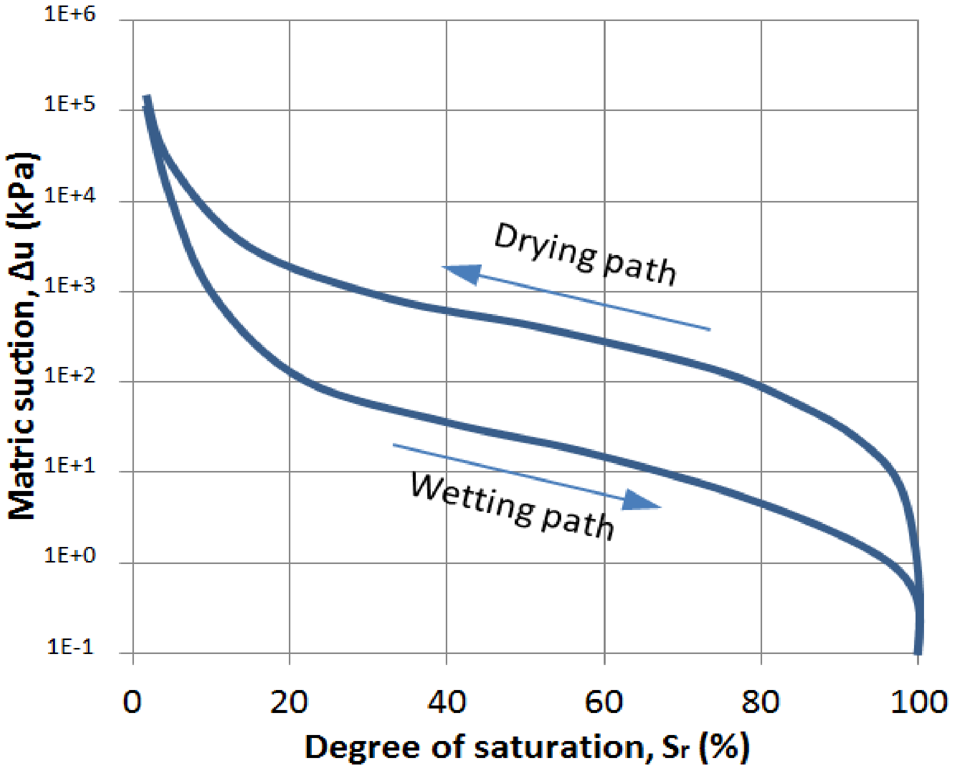
Water retention curve

- Relation Δu and S_R

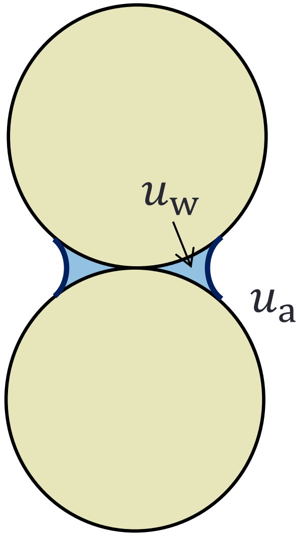
- Degree of saturation:

$$S_\mathrm{R} = \frac{V_\mathrm{w}}{V_\mathrm{v}}$$

- Hysteresis



Capillary bridge / meniscus



- Suction:

$$\Delta u = u_{\mathrm{a}} - u_{\mathrm{w}}$$

- Capillary force:

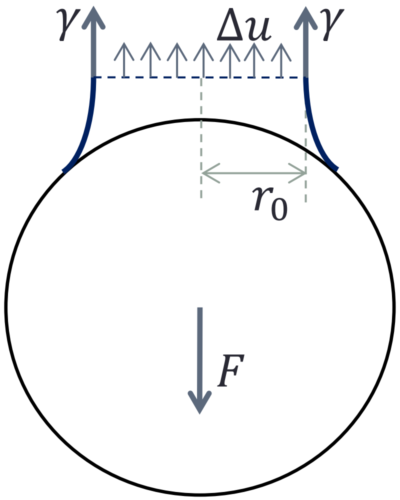
$$F_{\mathrm{cap}} = F_{\mathrm{L}} + F_{\mathrm{T}}$$

  - Laplace force

$$F_{\mathrm{L}} = \pi {r_0}^2 . \Delta u$$

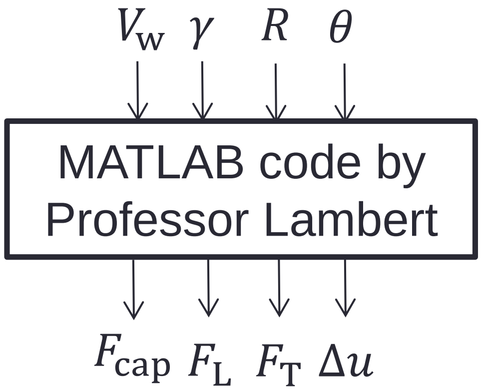
  - Tension force

$$F_{\mathrm{T}} = 2\pi r_0 . \gamma$$



Capillary force - determination

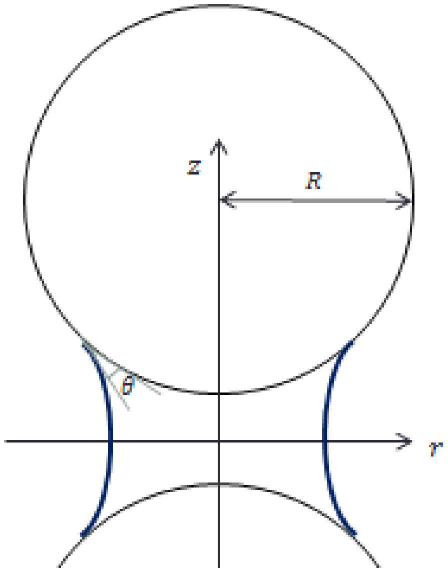
- Determination of the shape $r(z)$



V_{w} γ R θ

MATLAB code by
Professor Lambert

F_{cap} F_{L} F_{T} Δu



- Israelachvili
  - Small volumes of water:

$$F = 2\pi R\gamma\cos\theta$$

Comparison

Effective stress

- Bishop
  - $\sigma' = \sigma + \chi \Delta u$
  - until now: $\chi = S_\mathrm{R}$

$$\chi = ?$$

Capillary force

- Determination of the shape
  - $F_\mathrm{cap} = \pi . r_0{}^2 . \Delta u + 2\pi r_0 . \gamma$
- Israelachvili
  - $F = 2\pi R\gamma \cos\theta$

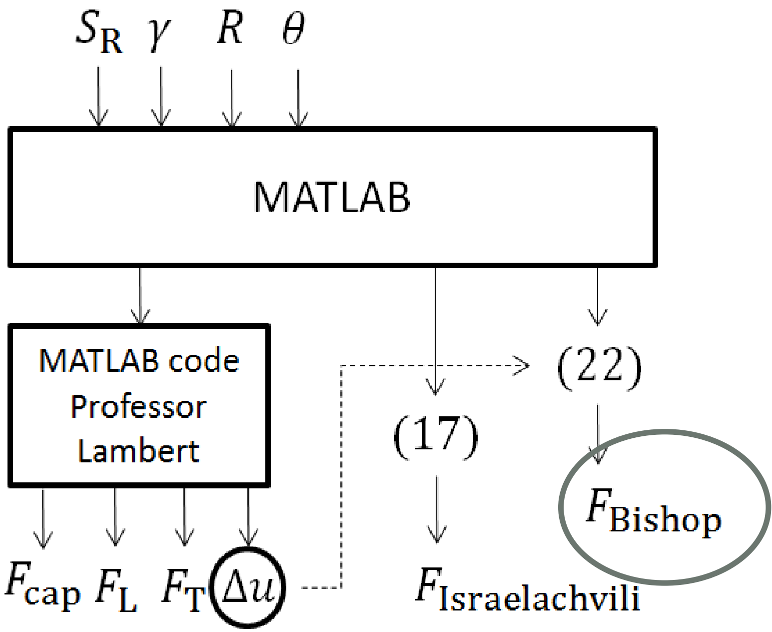
ANALYTICAL COMPARISON

Effective stress parameter

Comparison

Effective stress

- Bishop
  - $\sigma' = \cancel{\sigma} + \chi \Delta u$



S_R γ R θ

MATLAB

MATLAB code Professor Lambert

F_{cap} F_L F_T Δu

(17)

(22)

$F_{Israelachvili}$

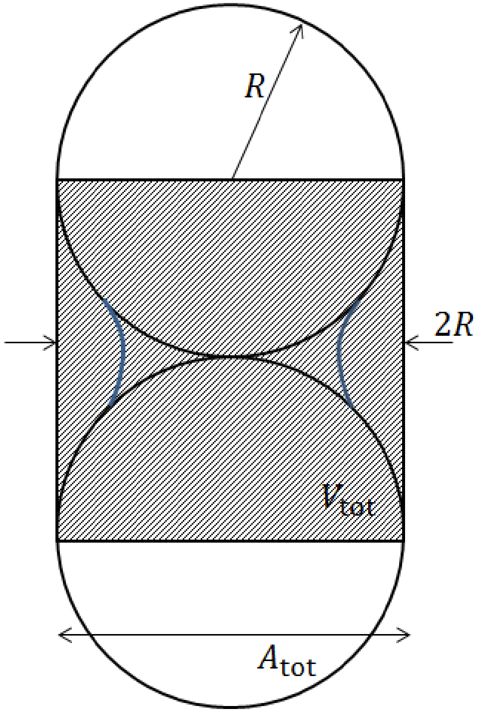
F_{Bishop}

Capillary force

- Determination of the shape
  - $F_{cap} = \pi . r_0{}^2 . \Delta u + 2\pi r_0 . \gamma$
- Israelachvili
  - $F = 2\pi R \gamma \cos\theta$

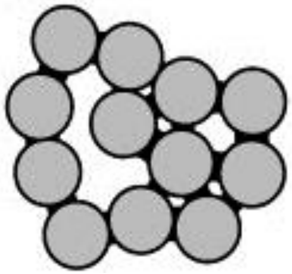
Conversion

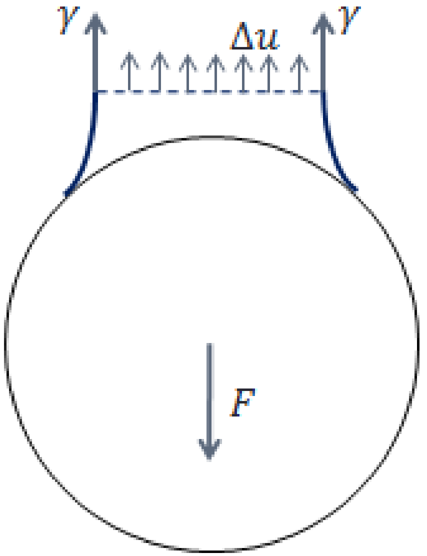
- F_{cap} vs σ'
  - $\sigma' = \frac{F_{\mathrm{cap}}}{A_{\mathrm{tot}}} = \frac{F_{\mathrm{cap}}}{\pi R^2}$
- V_{w} vs S_{R}
  - $S_{\mathrm{R}} = \frac{V_{\mathrm{w}}}{V_{\mathrm{v}}} = \frac{V_{\mathrm{w}}}{V_{\mathrm{tot}} - 2.\frac{1}{2}V_{\mathrm{sphere}}} = \frac{V_{\mathrm{w}}}{\pi.R^2.2.R - \frac{4}{3}.\pi.R^3} = \frac{3.V_{\mathrm{w}}}{2.\pi.R^3}$



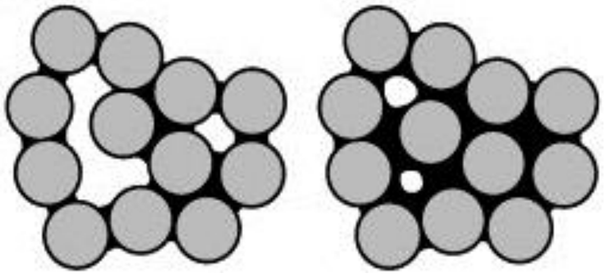
Pendular regime

- Inside regime ($S_R < 5\%$)
  - $F_{\text{cap}} = F_L + F_T$
    $= \pi . {r_0}^2 . \Delta u + 2\pi r_0 . \gamma$





- Outside regime ($S_R > 5\%$)
  - $F_{\text{cap}} = F_L + \cancel{F_T}$
    $= \pi . {r_0}^2 . \Delta u + 2\pi \cancel{r}_0 . \gamma$

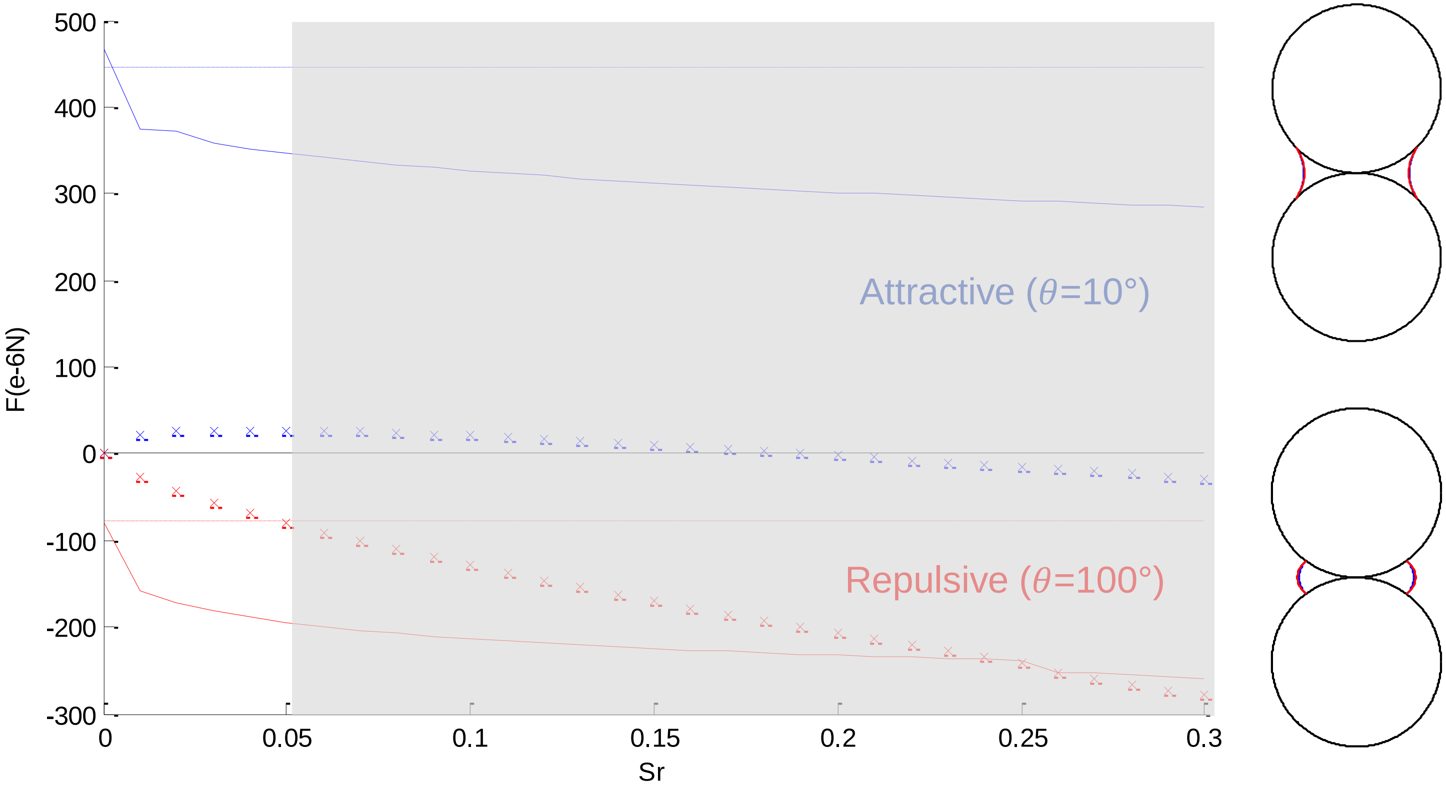


Inside regime

Shape: $F_{\mathrm{cap}} = \pi . {r_0}^2 . \Delta u + 2\pi r_0 . \gamma$

Israelachvili: $F = 2\pi R\gamma\cos\theta$

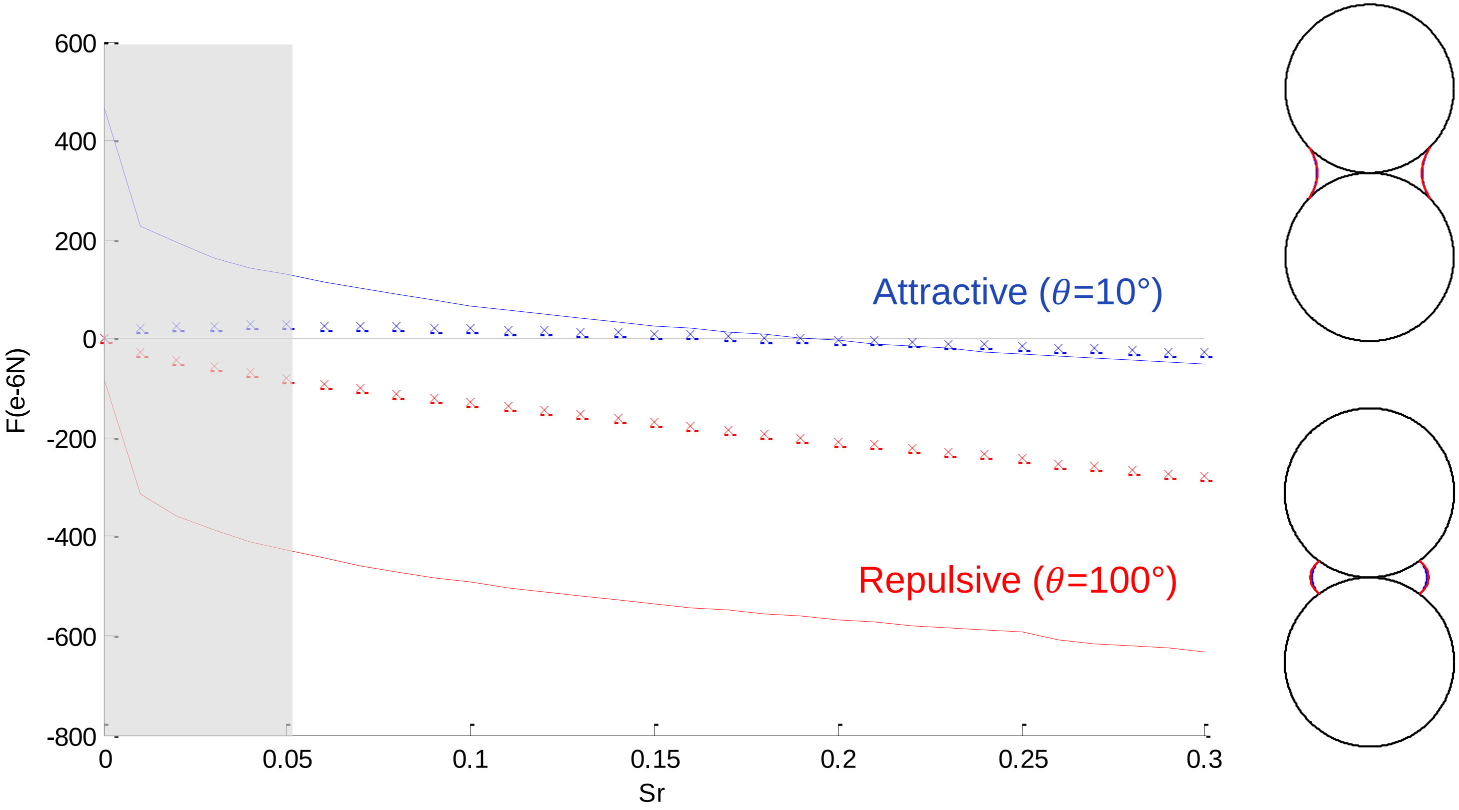
Bishop: $\sigma' = S_{\mathrm{R}}\Delta u$



Outside regime

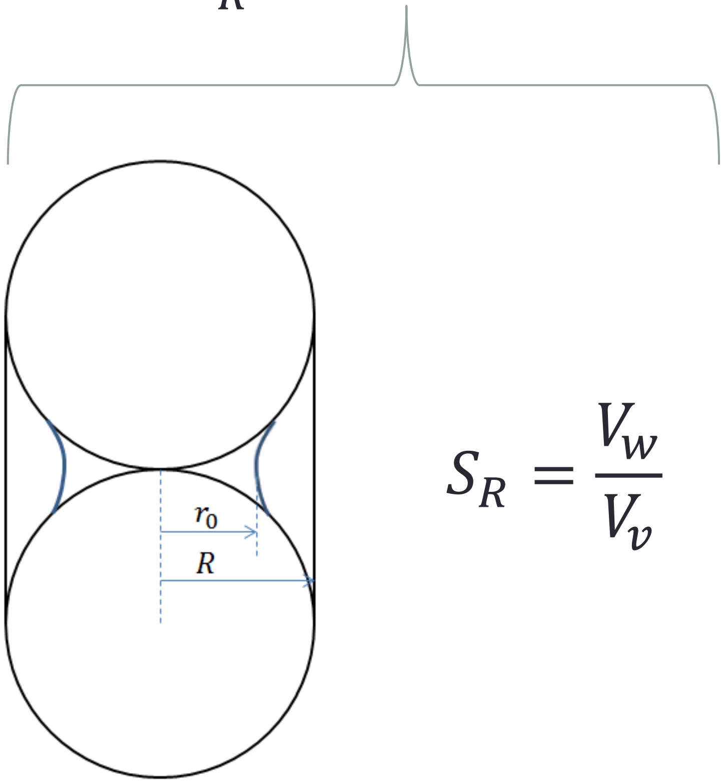
Shape: $F_{\text{cap}} = \pi . {r_0}^2 . \Delta u + \cancel{2\pi r_0} . \gamma$

Bishop: $\sigma' = S_{\text{R}} \Delta u$



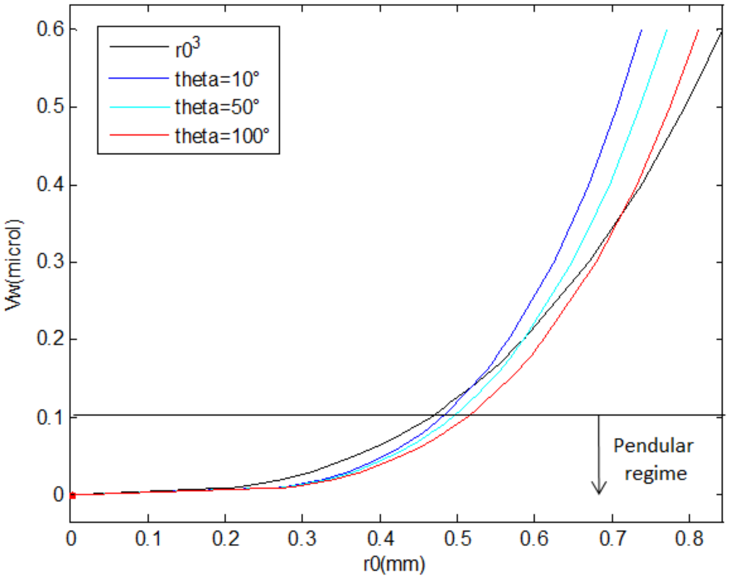
Bishop

- $F_{cap} = \pi {r_0}^2 . \Delta u + 2\cancel{\pi r}_0 . \gamma$
- $\sigma' = \frac{F_{cap}}{A_{tot}} = \frac{\pi {r_0}^2}{\pi R^2} . \Delta u = \frac{{r_0}^2}{R^2} . \Delta u \approx S_R . \Delta u$



$$S_R = \frac{V_w}{V_v}$$

Own approximation



- $V_w \approx r_0^3$
- $S_R = \frac{3.V_w}{2.\pi.R^3} \approx \frac{3.r_0^3}{2.\pi.R^3}$
- $F_{cap} = \pi {r_0}^2.\Delta u + 2\pi r_0.\gamma$
- $\sigma' = \frac{\pi {r_0}^2}{\pi R^2}.\Delta u + \frac{2\pi r_0.\gamma}{\pi R^2}$

$$\approx \left(\frac{2\pi}{3} S_R\right)^{2/3}.\Delta u + \frac{2\gamma}{R}\left(\frac{2\pi}{3} S_R\right)^{1/3}$$

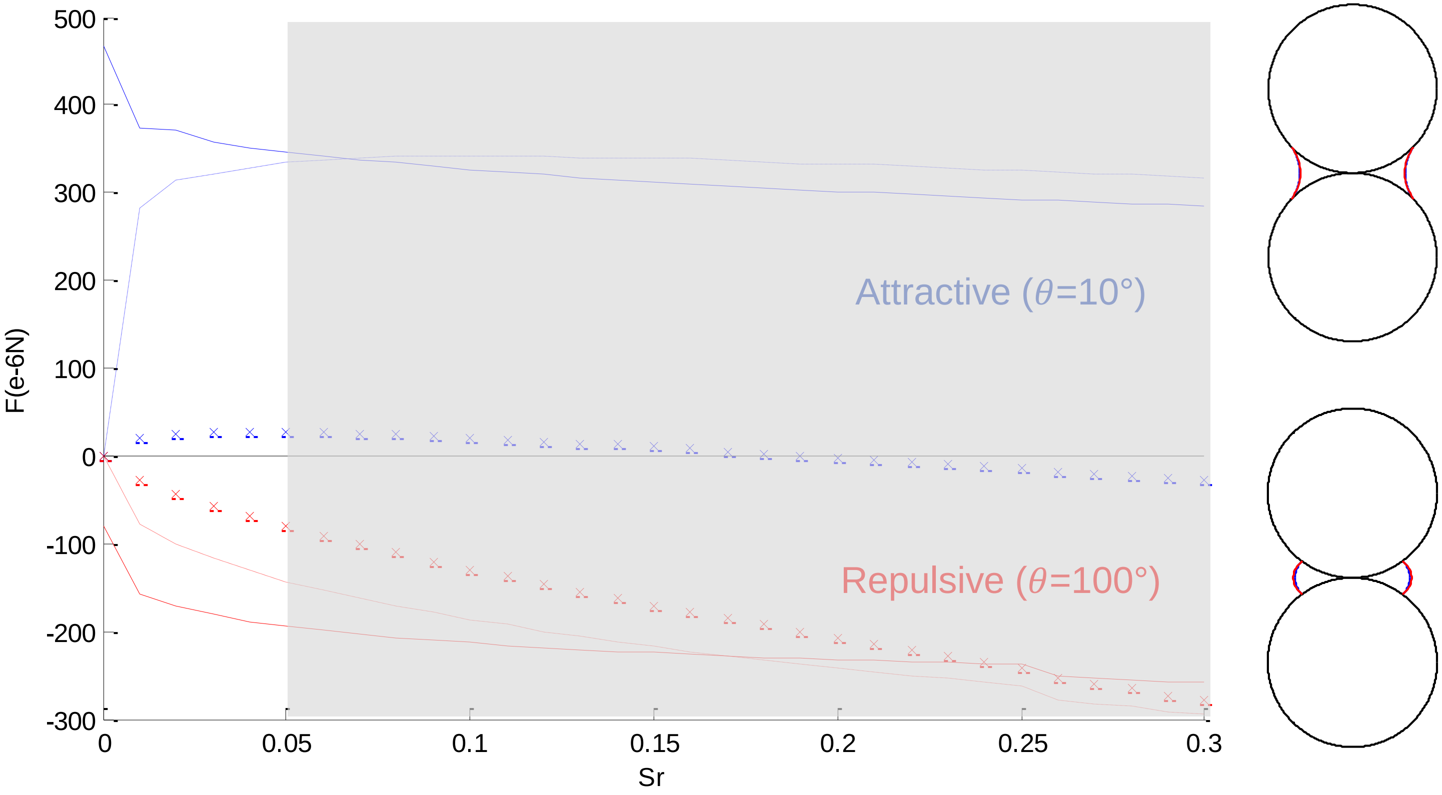
$$= \chi_1.\Delta u + \chi_2$$

Inside regime

Shape: $F_{\mathrm{cap}} = \pi . {r_0}^2 . \Delta u + 2\pi r_0 . \gamma$

Own approximation: $\sigma' = \chi_1 . \Delta u + \chi_2$

Bishop: $\sigma' = S_{\mathrm{R}} \Delta u$

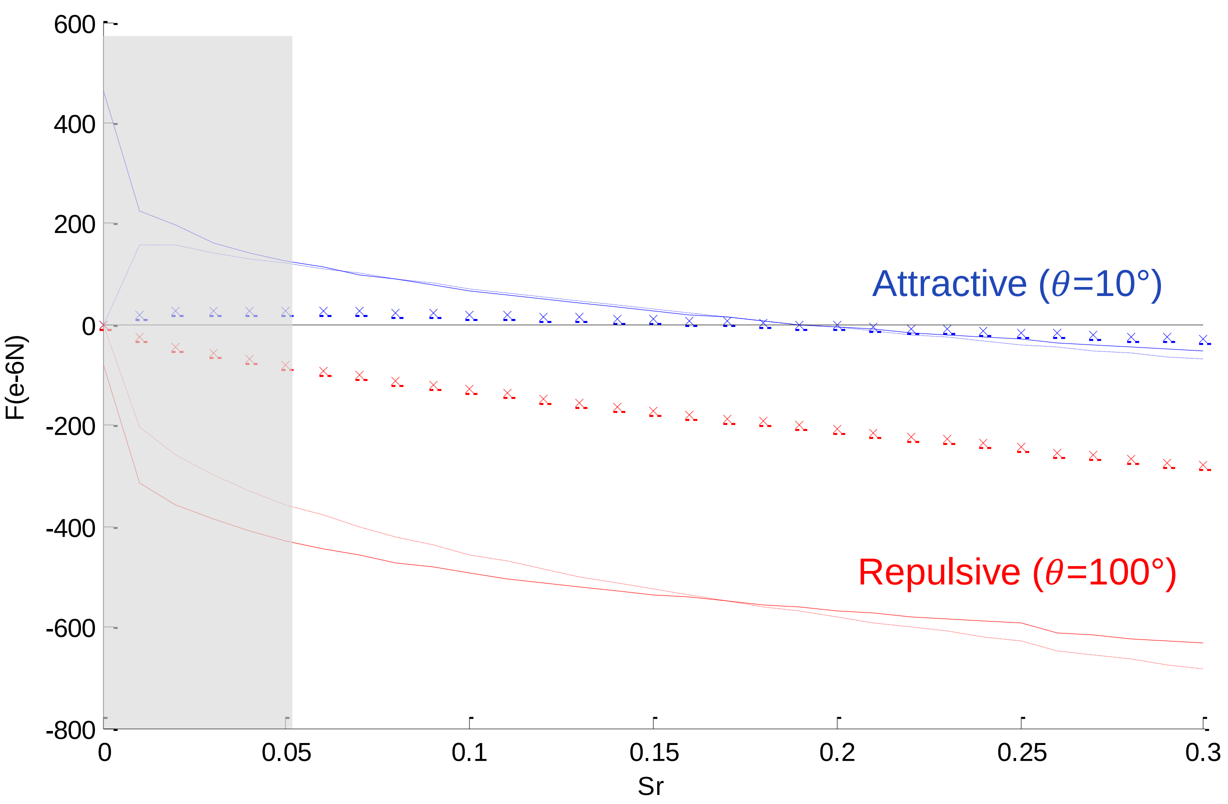


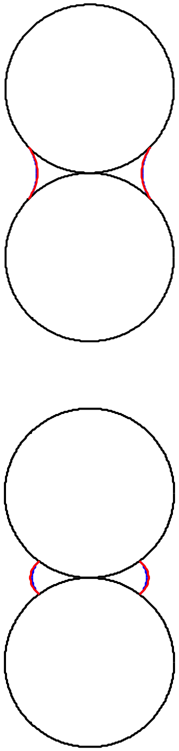
Outside regime

— Shape: $F_{\mathrm{cap}} = \pi . r_0{}^2 . \Delta u + \cancel{2\pi r_0 . \gamma}$

┈ Own approximation: $\sigma' = \chi_1 . \Delta u + \cancel{\chi_2}$

× × × Bishop: $\sigma' = S_{\mathrm{R}} \Delta u$

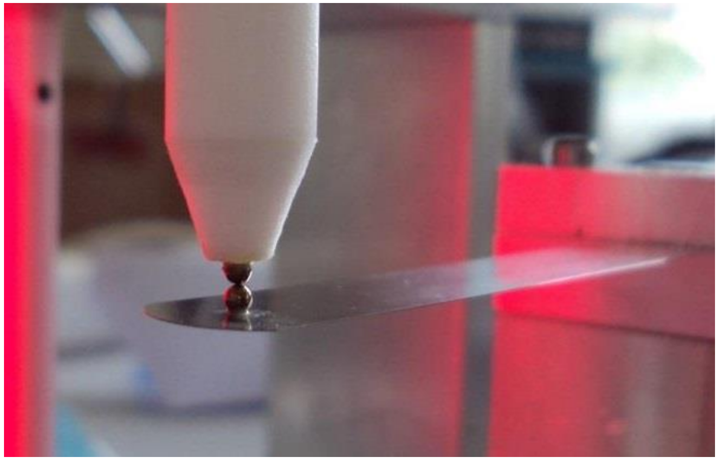


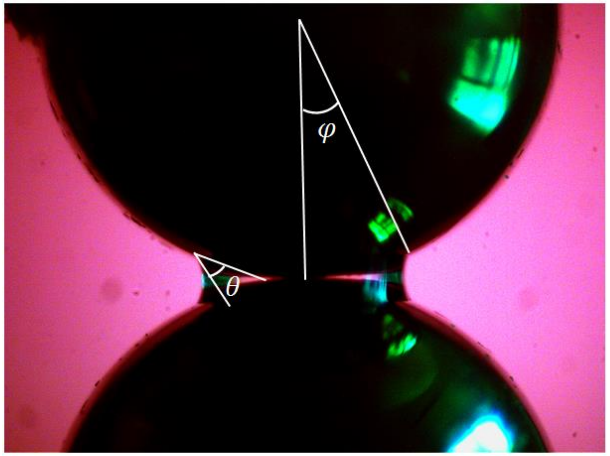


MICROSCOPIC TESTS

Capillary force

Test setup





- Measured:
  - Capillary force F_{cap}
  - Volume of water V_{w}
  - Contact angle θ
  - Filling angle φ

- Results:
  - Experiments = theory
  - Influence of θ less than expected

MACROSCOPIC TESTS

Shear strength

Shear strength

- Unsaturated soil:

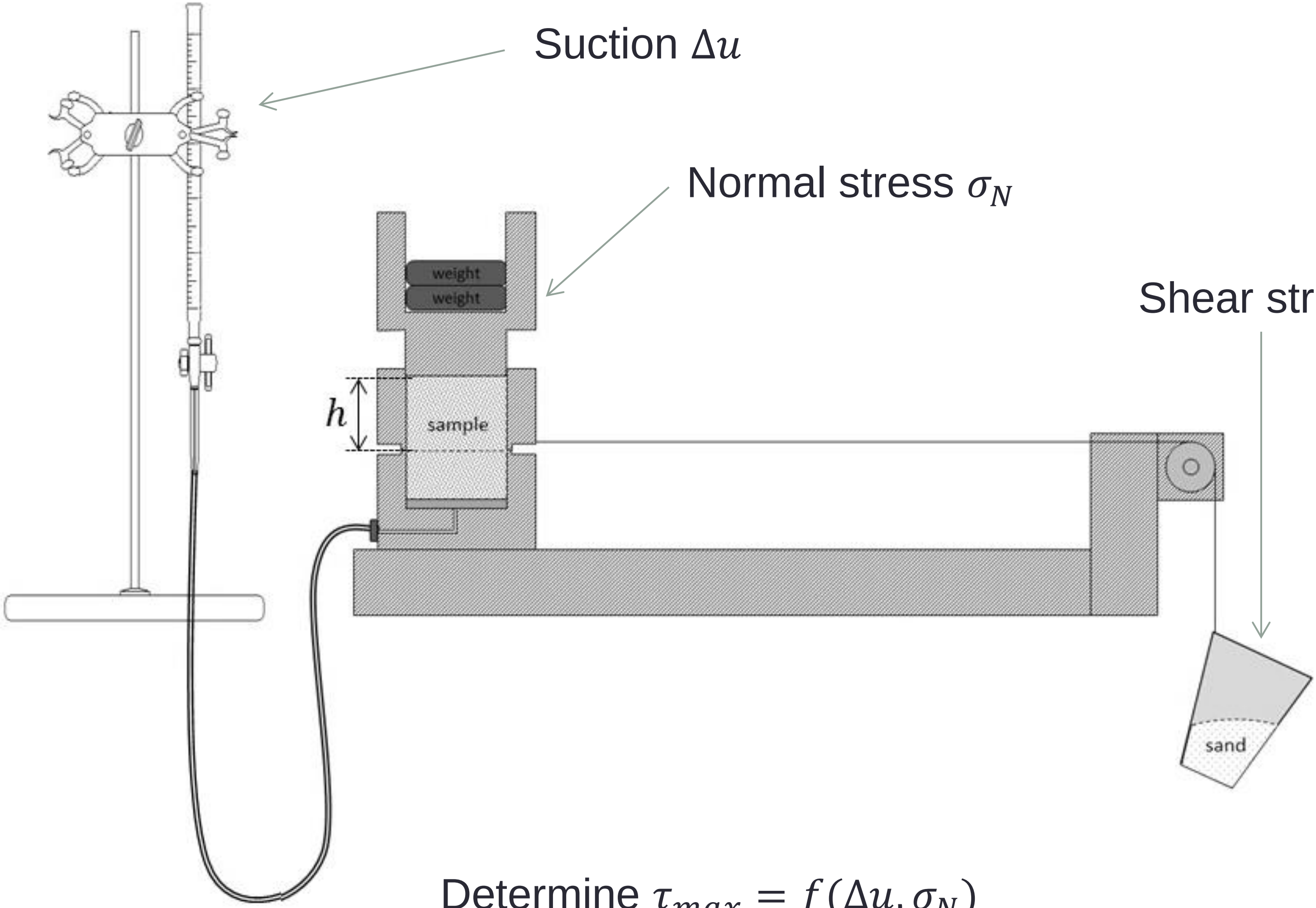
$$\tau_{\mathrm{max}} = c + \sigma_{\mathrm{N}}{}'.\tan\varphi$$

- Compare 4 possibilities:

No suction	$\sigma'_N = \sigma_N$
Bishop	$\sigma'_N = \sigma_N + S_R \Delta \mathrm{u}$
Own approximation 1	$\sigma'_N = \sigma_N + \chi_1 \Delta \mathrm{u}$
Own approximation 2	$\sigma'_N = \sigma_N + \chi_1 \Delta \mathrm{u} + \chi_2$

with $\chi_1 = \left(\frac{2\pi}{3} S_R\right)^{2/3}$ and $\chi_2 = \frac{2\gamma}{R}\left(\frac{2\pi}{3} S_R\right)^{1/3}$.

Test setup



Suction Δu

Normal stress σ_N

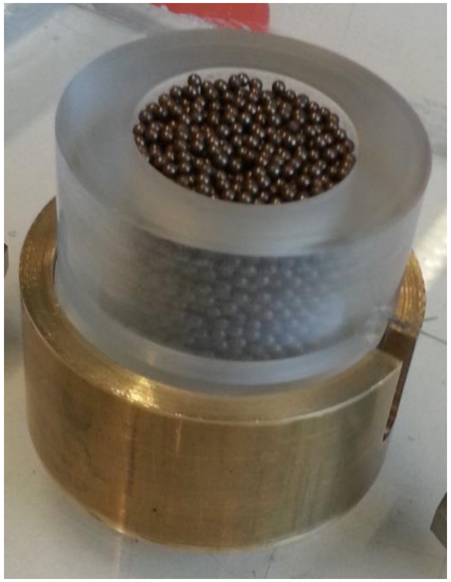
Shear stress τ

Determine $\tau_{max} = f(\Delta u, \sigma_N)$

Materials

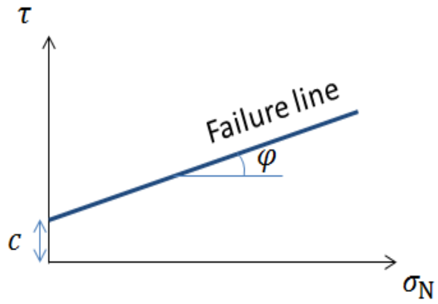
- Steel spherical particles
  - Diameter: 2mm

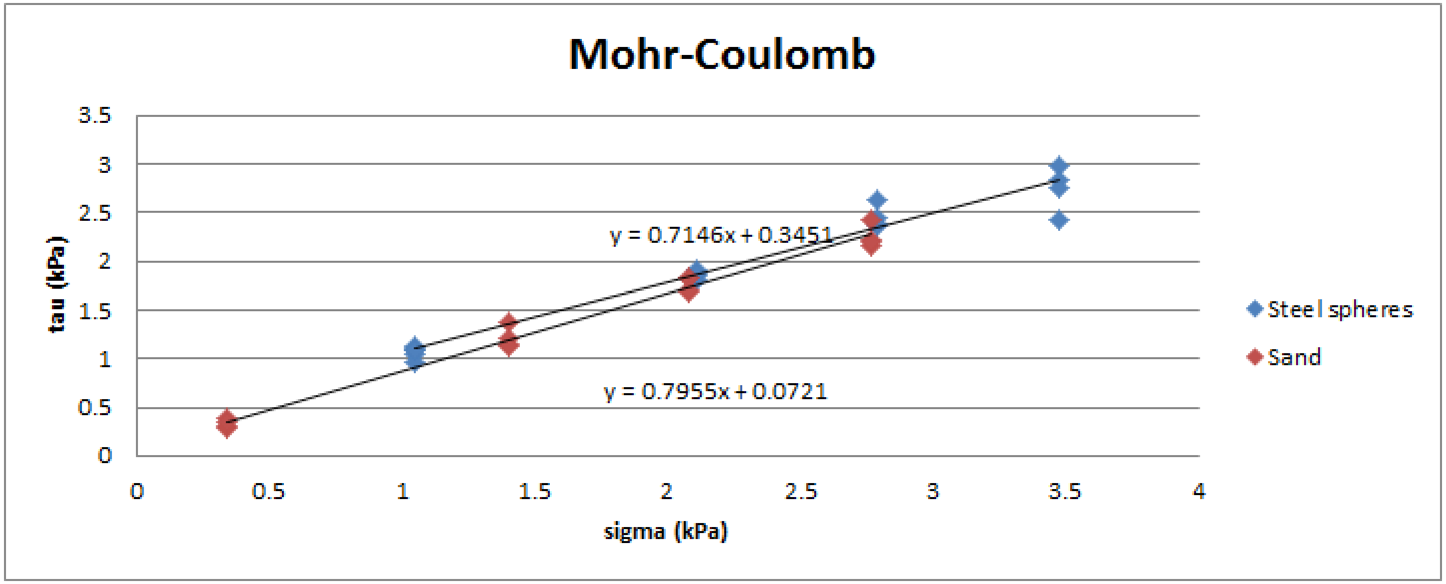
- Sieved sand
  - Diameter: 208-417µm



Dry tests

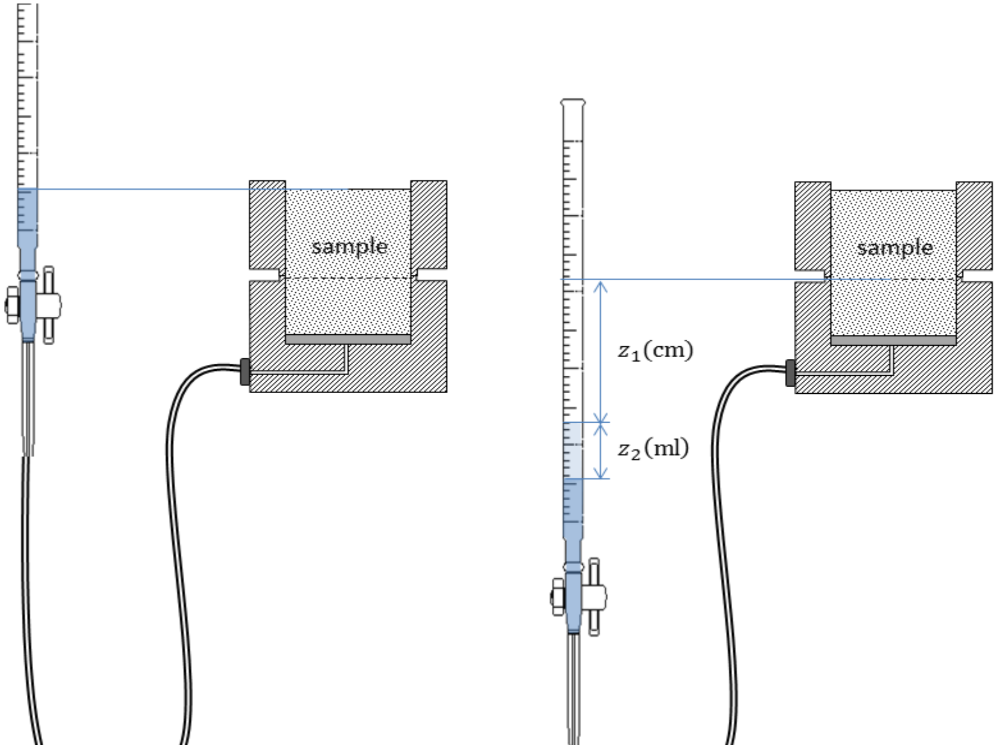
$$\tau_{\max} = c + \sigma_N . \tan\varphi$$





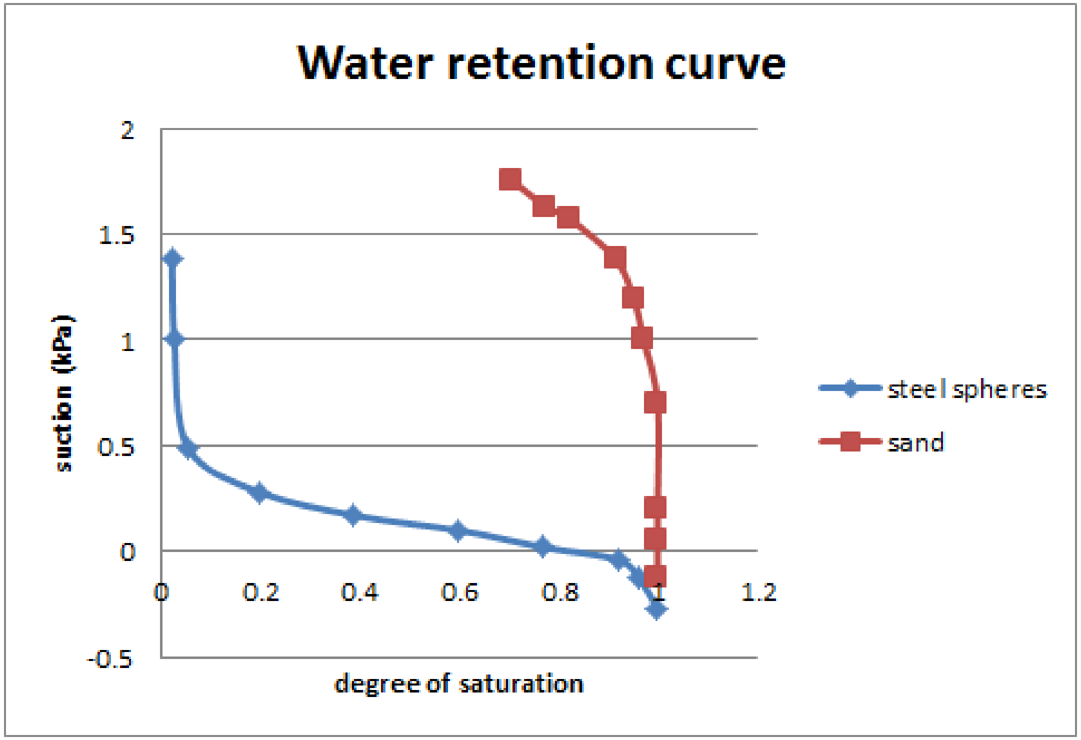
	Steel spheres	Sand
Cohesion c (kPa)	0.3452	0.0721
Friction angle φ	$\tan^{-1}(0.7146) = 35.55°$	$\tan^{-1}(0.7955) = 38.50°$

Apply suction



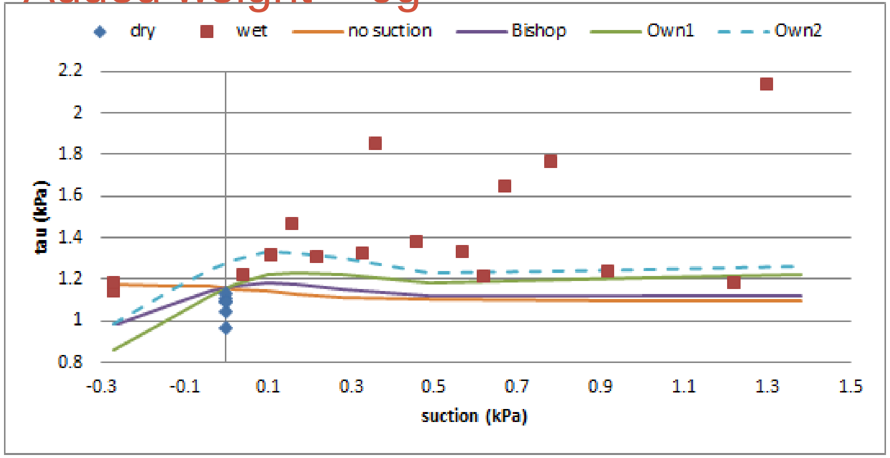
- $S_R = \frac{V_w}{V_v} = \frac{V_v - V_a}{V_v} = \frac{V_v - z_2}{V_v}$
- $\Delta u = z_1 . 10\text{kPa/m} = z_1 . 0.1\text{kPa/cm}$

Water retention curve

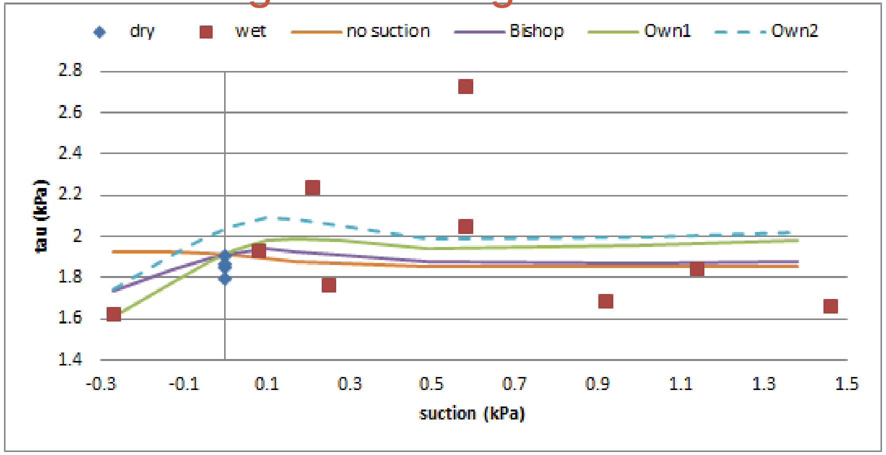


shear strength - steel spheres

Added weight = 0g



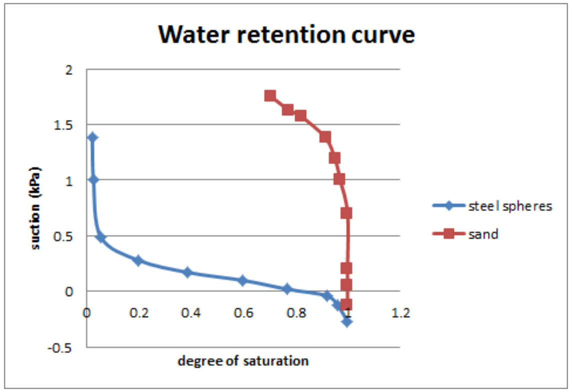
Added weight = 77.47g



$$\tau_{\max} = c + \sigma_{\mathrm{N}}{}'.\tan\varphi$$

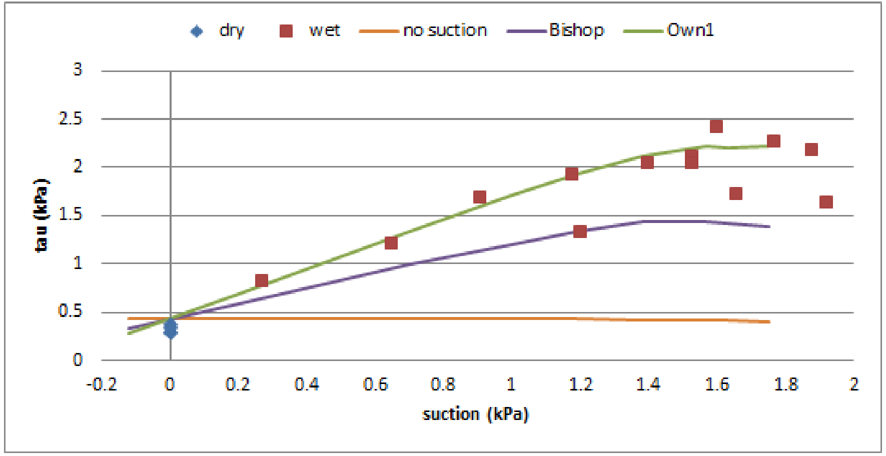
No suction	$\sigma'_N = \sigma_N$
Bishop	$\sigma'_N = \sigma_N + S_R \Delta u$
Own 1	$\sigma'_N = \sigma_N + \chi_1 \Delta u$
Own 2	$\sigma'_N = \sigma_N + \chi_1 \Delta u + \chi_2$

with $\chi_1 = \left(\frac{2\pi}{3} S_R\right)^{2/3}$ and $\chi_2 = \frac{2\gamma}{R}\left(\frac{2\pi}{3} S_R\right)^{1/3}$.

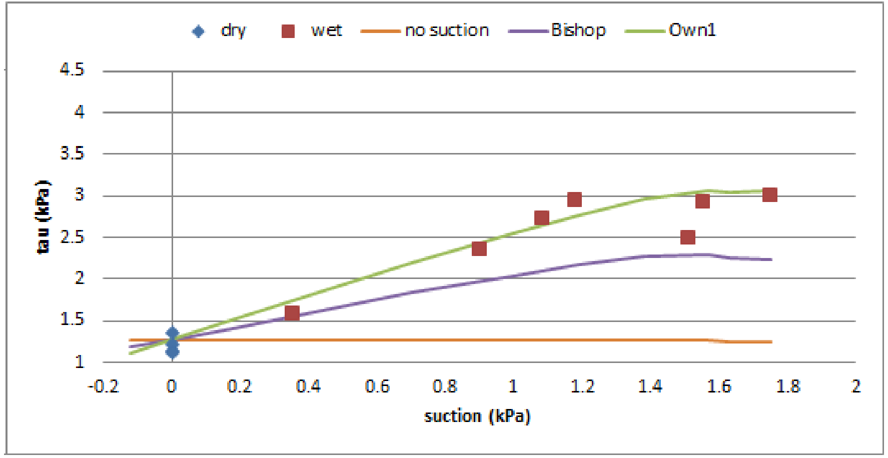


shear strength - sand

Added weight = 0g

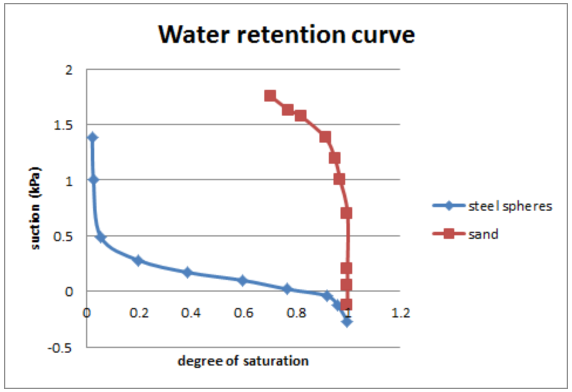


Added weight = 77.47g



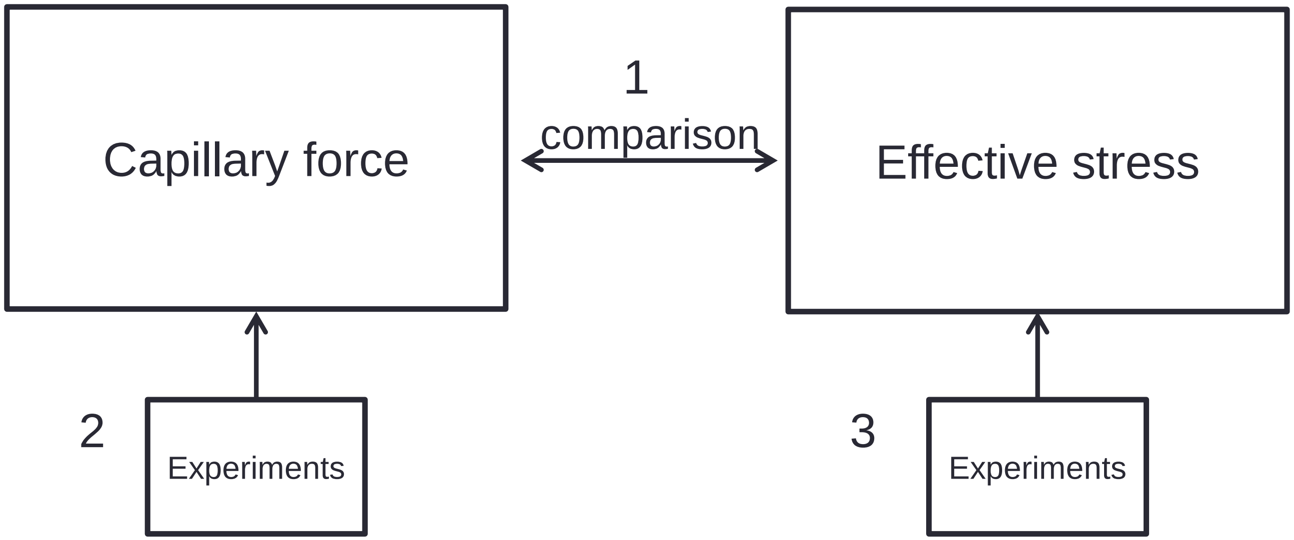
$$\tau_{\max} = c + \sigma_N{}'.\tan\varphi$$

No suction	$\sigma_N' = \sigma_N$
Bishop	$\sigma_N' = \sigma_N + S_R \Delta u$
Own 1	$\sigma_N' = \sigma_N + \chi_1 \Delta u$
Own 2	$\sigma_N' = \sigma_N + \chi_1 \Delta u + \chi_2$



CONCLUSION

Overview



Capillary force

1

comparison

Effective stress

2

Experiments

3

Experiments

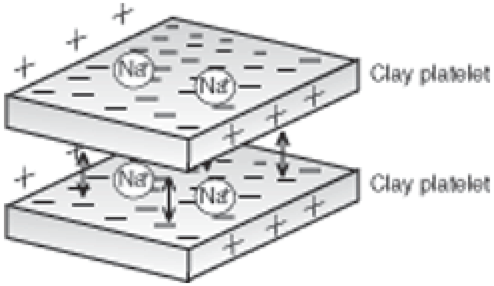
Conclusion

$$\sigma_{\mathrm{N}}{}' = \sigma_{\mathrm{N}} + \chi \Delta u$$

$$\chi = S_{\mathrm{R}} \;<\; \chi = \left(\frac{2\pi}{3} S_R \right)^{2/3}$$

Perspective

- Polydisperse soils
  - $V_w \approx r_0^3$?
- Clay particles
  - Platelets
  - Chemical interaction



THANK YOU