# Numerical modeling of cracking process in partially saturated porous media and application to rainfall-induced slope instability analysis

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# Outline

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PART ONE

# BACKGROUND

# Introduction

**Landslide**

| | | |
|---|---|---|
| Predisposing factors | • Lithology<br>• Faults<br>• Land use | Easily assessed by spatial analysis techniques *(Ayalew et al. 2005)* |
| Triggering factors | • Rainfall<br>• Snowmelt<br>• Earthquakes | Difficult to estimate at a regional scale *(Griffiths 2014)* |

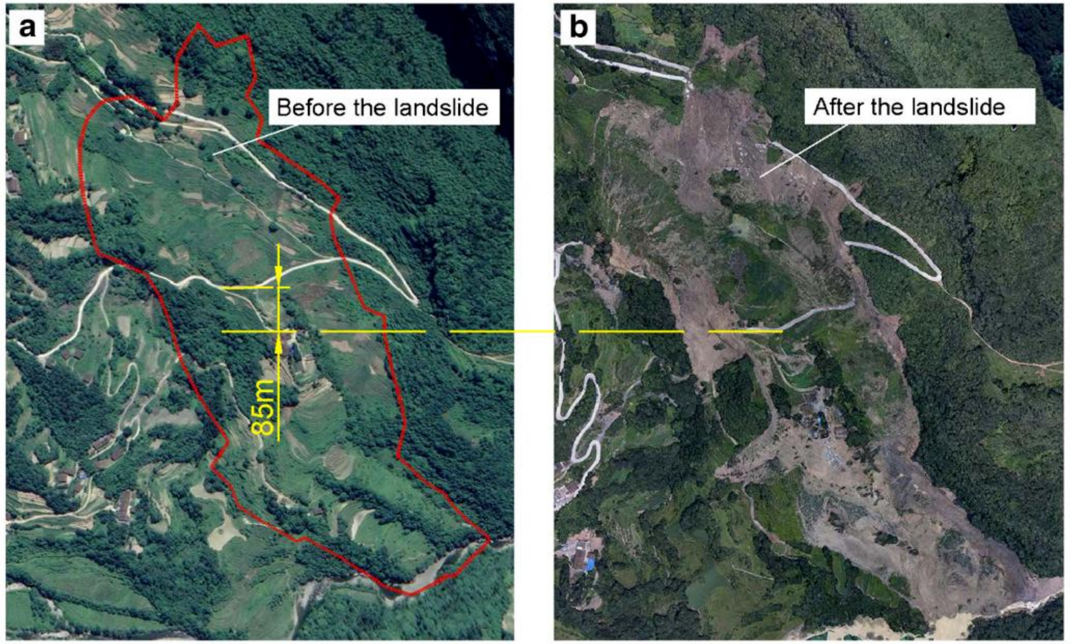


*Figure –Zhongbao lanslide at Wulong, China*
*(CHEN et al. 2021)*

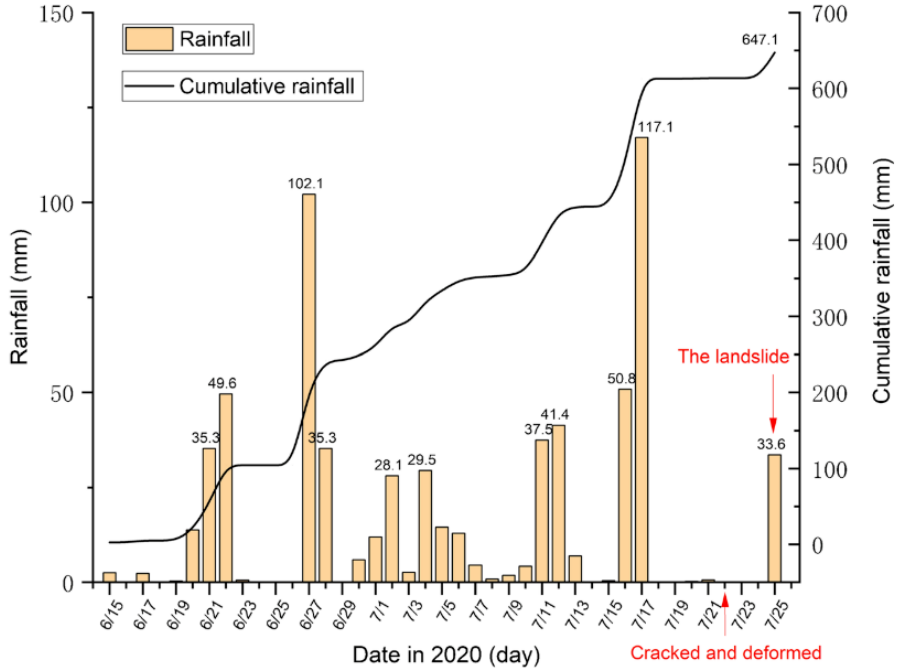


*Figure –Rainfall monitoring data*
*(CHEN et al. 2021)*

# Objectives

Triggering factors: **Rainfall** → Material weakening Induced deformation → Crack evolution

Approaches for slope stability analysis:

Limit equilibrium method (LEM) → Safety factor

Numerical modelling, progressive deformation and instability e.g. Finite element method (FEM) → Safety factor

# Methodology: Phase-field Method (PFM)

Regularized crack topology:

$$A_\Gamma = \int_\Gamma \mathrm{d}A \cong \int_\Omega \gamma_d(d, \nabla d)\, \mathrm{d}V$$

- Predict not only **crack initiation** but also the **crack propagation path**;
- Deal with merging and branching of **multiple cracks**;
- Easy to incorporate the **multi-field physics**

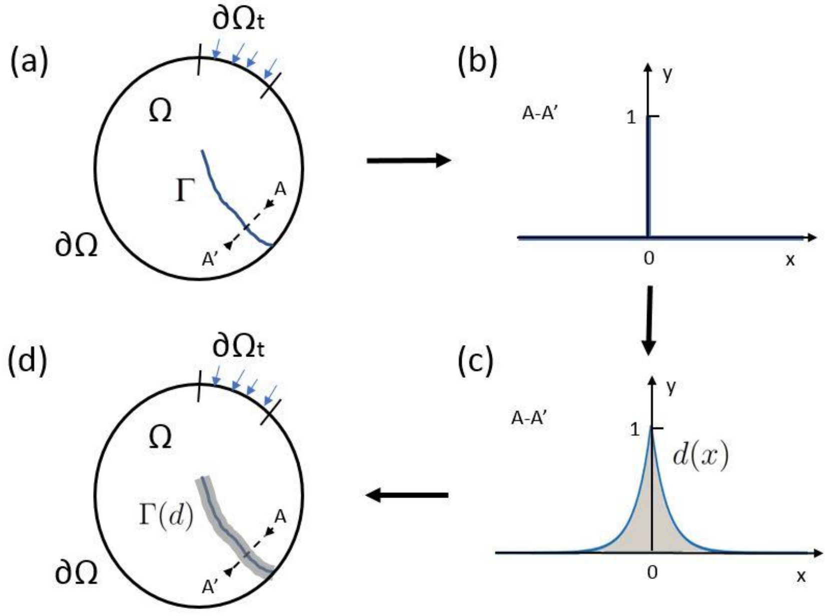


*Figure –Regularized crack topology*

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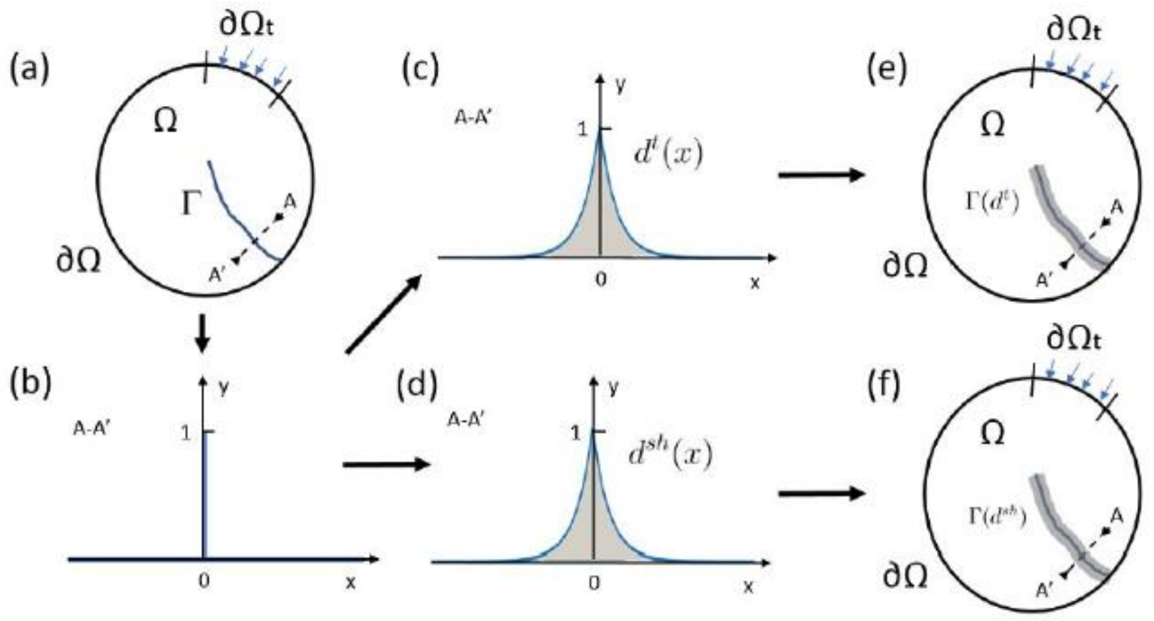
PART TWO

# PHASE-FIELD FORMULATIONS FOR PARTIALLY SATURATED MEDIA

# Regularized crack fields

Two independent variables $d^t$ and $d^s$ to approximate the crack surface area:




The total crack surface density : $\gamma_d(d,\nabla d) = \boxed{\gamma_d^t(d^t,\nabla d^t)} + \boxed{\gamma_d^s(d^s,\nabla d^s)}$

Tensile crack density $\dfrac{(d^t)^2}{2l_d} + \dfrac{l_d}{2}\nabla d^t.\nabla d^t$

$\dfrac{(d^s)^2}{2l_d} + \dfrac{l_d}{2}\nabla d^s.\nabla d^s$ Compressive-shear crack density

The energy dissipated by cracks : $\boxed{D(d^s,d^t) = \int_\Omega \left[g_c^t\gamma_d^t(d^t,\nabla d^t) + g_c^s\gamma_d^s(d^s,\nabla d^s)\right] dV}$

## Regularized crack fields

Poroelastic model for the undamaged material (Coussy, 2010) :

$$d\boldsymbol{\sigma}^0 = d\boldsymbol{\sigma}^{\mathrm{b}0} - bS_w dp_w I$$

$$dp_w = \mathrm{M}_{ww}\left[-bS_w d\boldsymbol{\varepsilon}_v + \left(\frac{dm_w}{\rho_w}\right)\right]$$

- The capillary pressure ():

$$p_c = -p_w$$

- The extended Biot's effective stress :

$$d\boldsymbol{\sigma}^{\boldsymbol{b}0} = d\boldsymbol{\sigma}^0 + bS_w dp_w I = \mathbb{C}^{b0} : d\boldsymbol{\varepsilon}$$

- The water saturation degree (van Genuchten, 1980) :

$$S_w = S_r + S_e(1 - S_r)$$

$$S_e = \left[1 + \left(\frac{p_c}{p_{\mathrm{cr}}}\right)^n\right]^{-m}$$

## The total energy functional of partially saturated cracked material

$$E(\boldsymbol{\varepsilon}, m_w, d^t, d^s) = \underbrace{\boxed{\int_\Omega \psi(\boldsymbol{\varepsilon}, m_w, d^t, d^s)\mathrm{d}V}}_{\text{stored energy}} + \underbrace{\boxed{\int_\Omega \mathcal{D}(d^t, d^s)\,\mathrm{d}V}}_{\text{cracking dissipation}}$$

$$\psi(\boldsymbol{\varepsilon}, m_w, d^t, d^s) = \underbrace{\psi_{eff}(\boldsymbol{\varepsilon}^e, d^t, d^s)}_{\text{Skeleton deformation}} + \underbrace{\psi_{fluids}(\boldsymbol{\varepsilon}^e, m_w, m_g)}_{\text{Fluid mass change}}$$

# Stored energy for partially saturated media with cracks

❑ The stored elastic energy of porous medium:

$$\psi_{eff}(\boldsymbol{\varepsilon}, d^t, d^s) = g(d^t)\ \boxed{W_+^b(\boldsymbol{\varepsilon})} + g(d^s)\ \boxed{W_-^b(\boldsymbol{\varepsilon})}$$

$$W_+^b(\boldsymbol{\varepsilon}) = \frac{1}{2}\boldsymbol{\sigma}_+^{\mathrm{b}} : \boldsymbol{\varepsilon}$$

Tensile crack driving energy

$$W_-^b(\boldsymbol{\varepsilon}) = \frac{1}{2}\boldsymbol{\sigma}_-^{\mathrm{b}} : \boldsymbol{\varepsilon}$$

Shear crack driving energy

- The degradation function (Miehe et al. 2010) :

$$g(d^\alpha) = (1 - d^\alpha)^2$$

- The Decomposition of effective stress tensors :

$$\boldsymbol{\sigma}_\pm^b = \sum_{a=1}^{3} \langle \sigma_a \rangle_\pm \boldsymbol{n}_a \otimes \boldsymbol{n}_a$$

❑ The energy due to fluid mass change :

$$\psi_{fluids}(\boldsymbol{\varepsilon}, m_w, d^t, d^s) \equiv \psi_{fluids}(\boldsymbol{\varepsilon}, m_w) = \frac{1}{2} M_{ww}\left[ bS_w \boldsymbol{\varepsilon}_{\mathrm{v}} - \left(\frac{m}{\rho}\right)_w \right]^2$$

$$\Pi\left(\dot{\boldsymbol{u}}, \dot{m}_w, \dot{m}_g, \dot{d}^t, \dot{d}^s\right) = \dot{E}\left(\dot{\boldsymbol{u}}, \dot{m}_w, \dot{m}_g, \dot{d}^t, \dot{d}^s\right) - \dot{P}_{ext}$$

## Governing equations for phase-field variables

$$-g_t'(d^t)\boxed{W_+^b} - g_c^t\left[\frac{d^t}{l} - l\,\mathrm{div}(\nabla d^t)\right] = 0$$

$$W_+^e = \frac{1}{2}\boldsymbol{\sigma}_+ : \boldsymbol{\varepsilon}^e$$

$$\mathcal{H}^t = \max_{t\in[0,t]} W_+^e$$

$$\boxed{-2(1-d^t)\mathcal{H}^t - g_c^t\left[\frac{d^t}{l} - l\,div(\nabla d^t)\right] = 0}$$

$$-g_s'(d^s)\boxed{W_-^b} - g_c^s\left[\frac{d^s}{l} - l\,\mathrm{div}(\nabla d^s)\right] = 0$$

$$W_-^s = \frac{1}{2G}\left\langle\frac{\sigma_3^- - \sigma_1^-}{2\cos\varphi} + \frac{\sigma_3^- + \sigma_1^-}{2}\tan\varphi - c\right\rangle_+^2$$

$$\mathcal{H}^s = \max_{t\in[0,t]} W_-^s$$

$$\boxed{-2(1-d^s)\mathcal{H}^s - g_c^s\left[\frac{d^s}{l} - l\,div(\nabla d^s)\right] = 0}$$

$$\Pi\left(\dot{\boldsymbol{u}}, \dot{m}_w, \dot{m}_g, \dot{d}^t, \dot{d}^s\right) = \dot{E}\left(\dot{\boldsymbol{u}}, \dot{m}_w, \dot{m}_g, \dot{d}^t, \dot{d}^s\right) - \dot{P}_{ext} = 0$$

## Hydro-mechanics coupling functions for partially saturated medium

$$p_w - p_{w0} = M_w\left[-b_w \boldsymbol{\varepsilon}\,\mathrm{I} + \frac{m_w}{\rho_w}\right]$$

Darcy's law and mass conservation

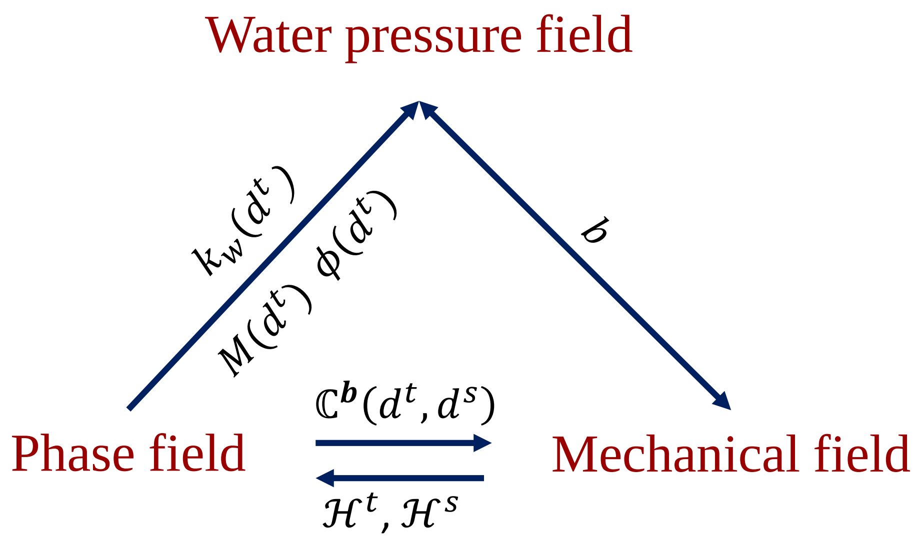
$$bS_w\dot{\varepsilon}_v + \frac{1}{M}\dot{p}_w = \frac{k_r k_w}{\mu_w}\cdot \operatorname{div}(\nabla p_w - \rho_w \vec{g})$$

$$\operatorname{div}(\boldsymbol{\sigma}) + \vec{f} = 0$$

$$\boldsymbol{\sigma} - \boldsymbol{\sigma}_0 = \mathbb{C}^b(d^t, d^s) : \boldsymbol{\varepsilon} - bS_w(p_w - p_{w0})\mathrm{I}$$

Effects of phase field on hydraulic parameters:

- Permeability: $k_w(d^t) = k_w^0 \exp(d^t)$
- Porosity: $\phi(d^t) = \phi^0 + (1-\phi^0)\, d^t$
- Scalar parameter: $\frac{1}{M(d^t)} = \frac{S_l^2\,[b-\phi(d^t)]}{K_s} + \frac{S_l\phi(d^t)}{K_f} - \phi(d^t)\frac{\partial S_l}{\partial p_c}$



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PART THREE

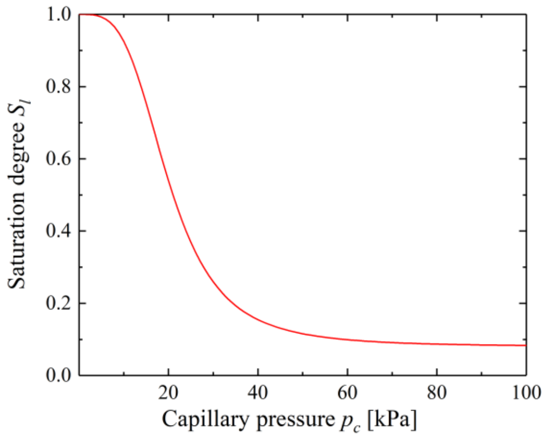
# NUMERICAL MODELING

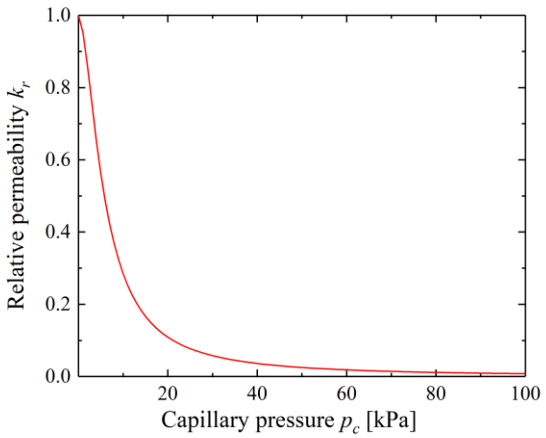
# Analysis of rainfall induced landslides

## Hydro-mechanical parameters:

| $\lambda$ | $\mu$ | $K_w$ | $\phi$ | $b$ | $k_{pl}$ |
|---|---|---|---|---|---|
| *2.9 GPa* | *0.7 GPa* | $2.2 \times 10^9\ Pa$ | *0.38* | *1.0* | $5 \times 10^{-12}\ m^2$ |

## Water retention and relative permeability curves




## Phase-field parameters:

| Critical energy $g_c^t$ | Critical energy $g_c^s$ | Crack length scale $l$ |
|---|---|---|
| *224 N/m* | *364 N/m* | *0.25m* |

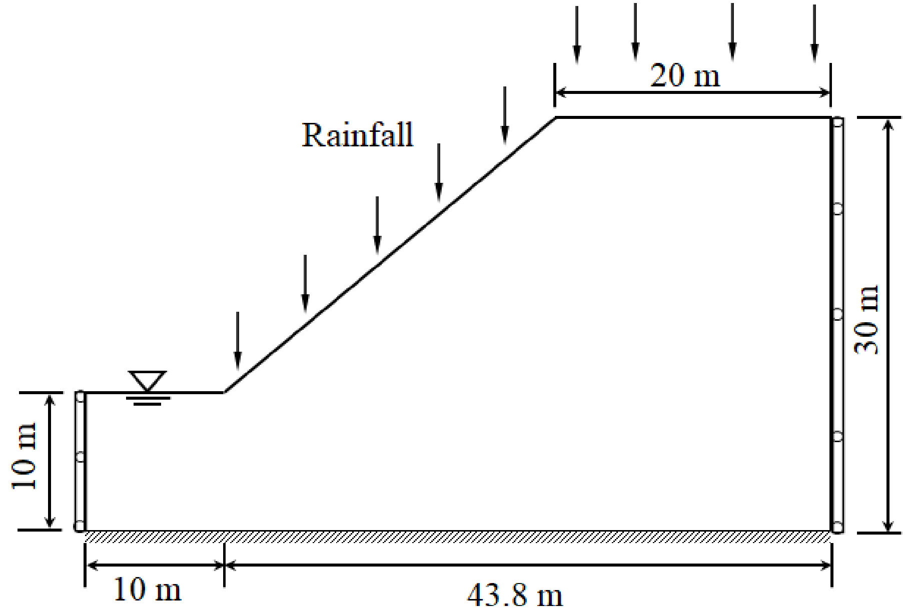



*Figure - Boundary conditions*

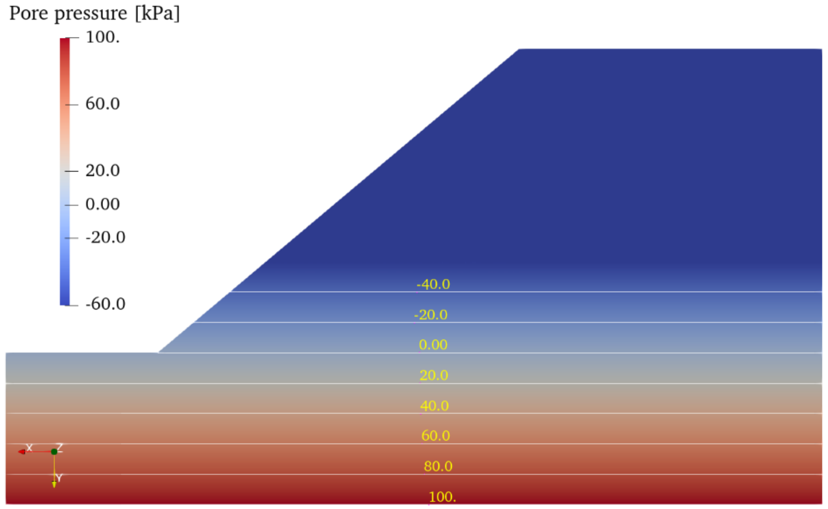



*Figure - Initial distribution of pore pressure*

# Analysis of rainfall induced landslides

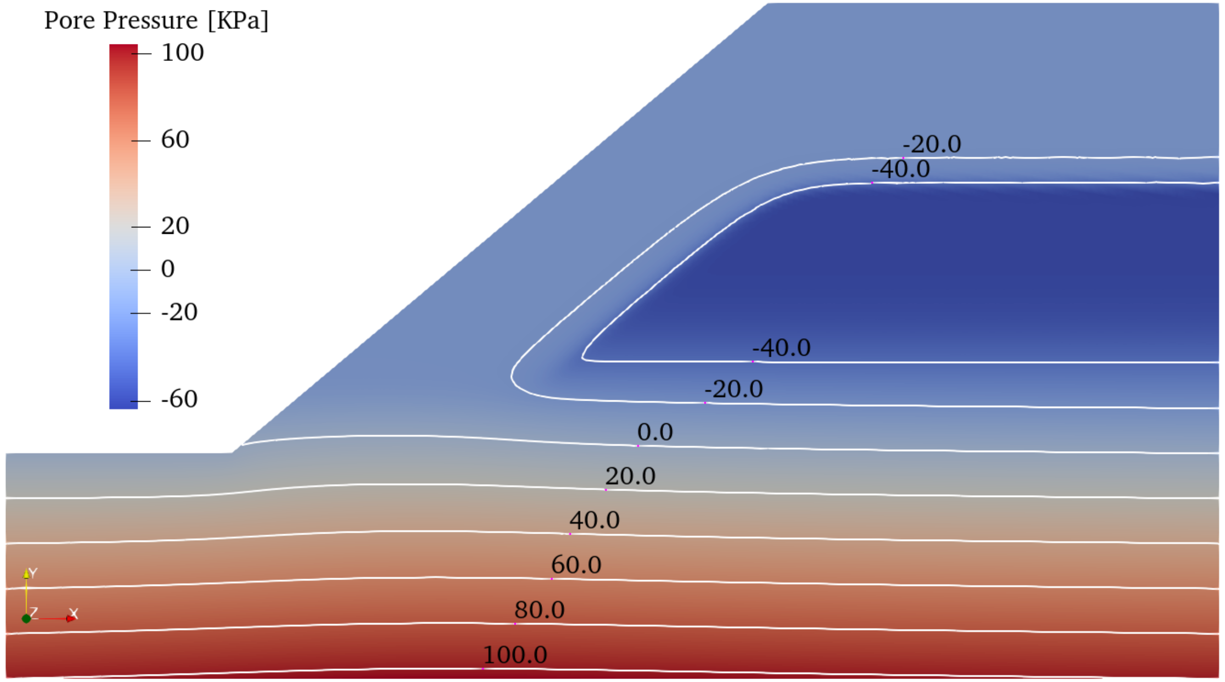



*Figure - Distribution of pore pressure without damage (after 66h)*

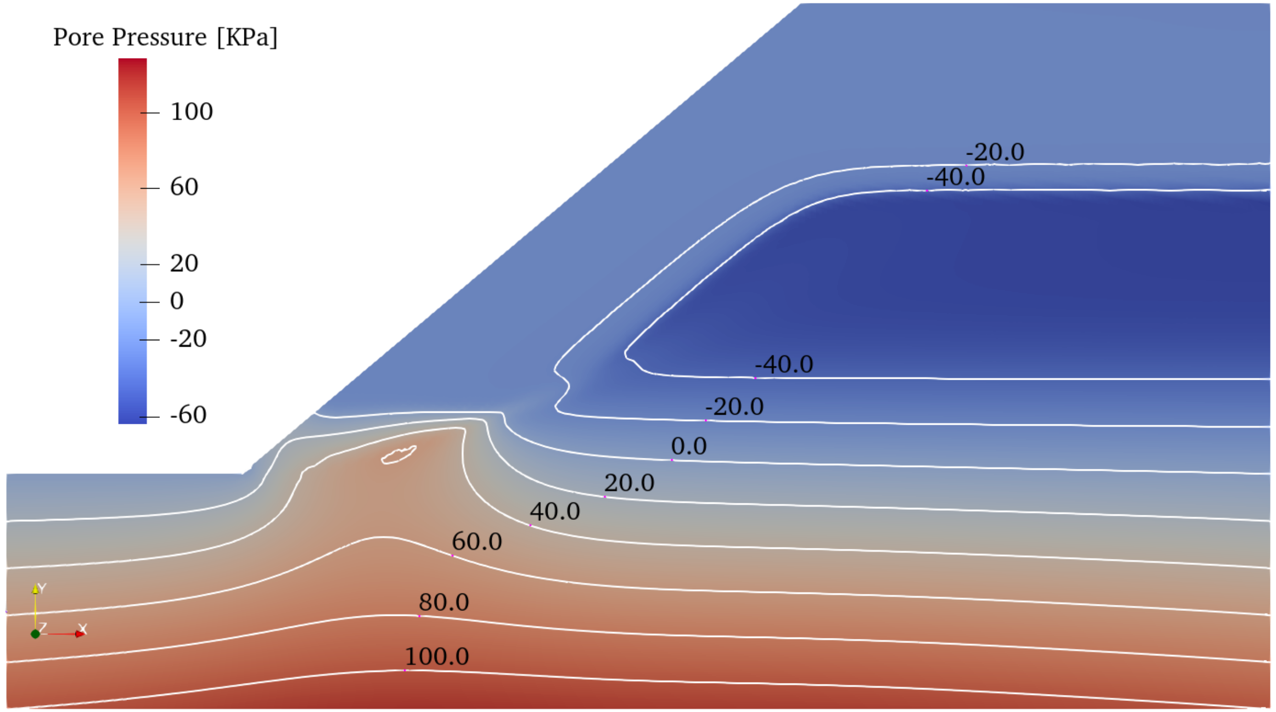



*Figure - Distribution of pore pressure when slope failure occurs (after 65.5h)*

Rainfall infiltration:

- Increment of underground water table
- Partially saturated ⟶ fully saturated (toe of the slope)

Pore pressure ⟷ cracks

# Analysis of rainfall induced landslides

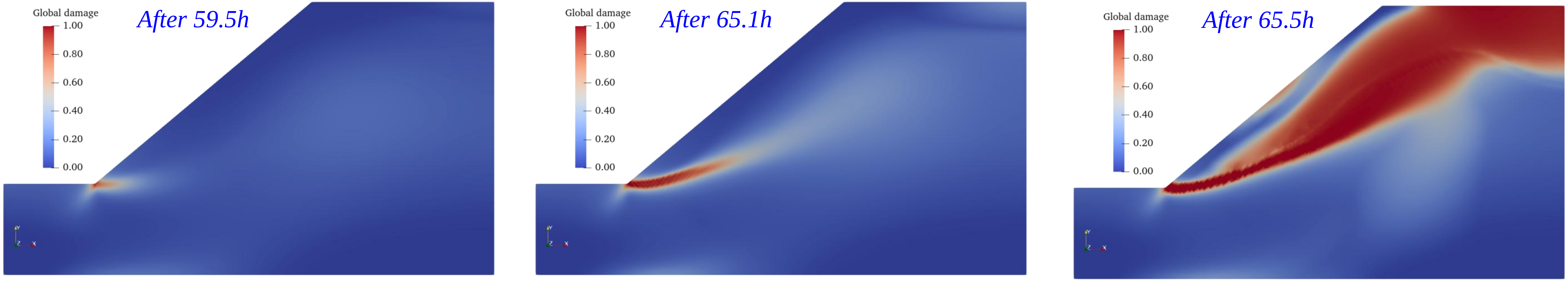



*Figure - Distribution of global damage*

- Onset of cracks: Around the toe of the slope
- Cracks path: Toe of slope ⟶ top of slope

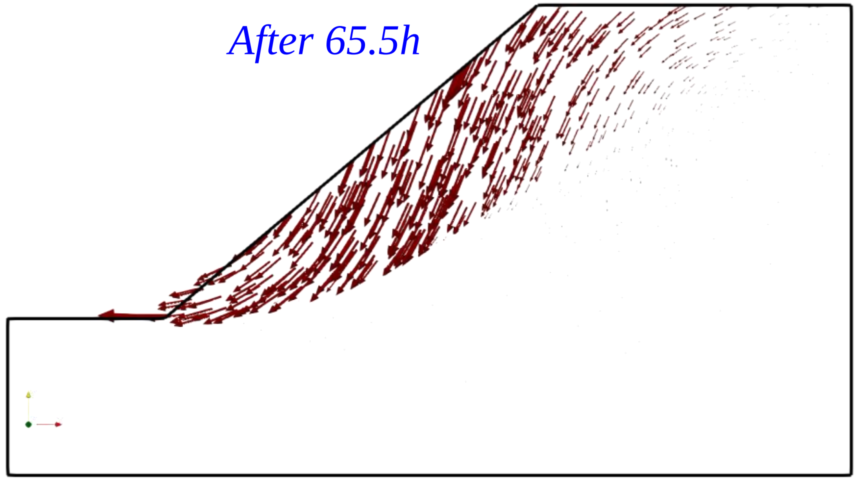
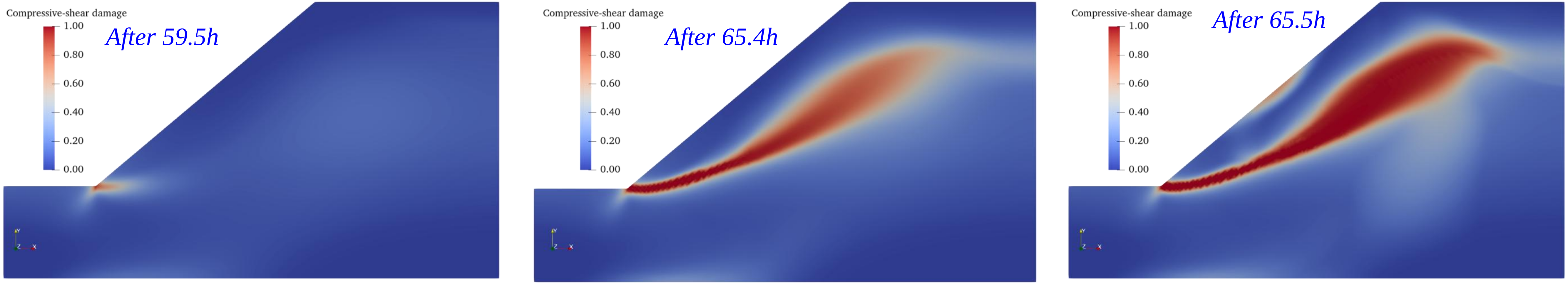
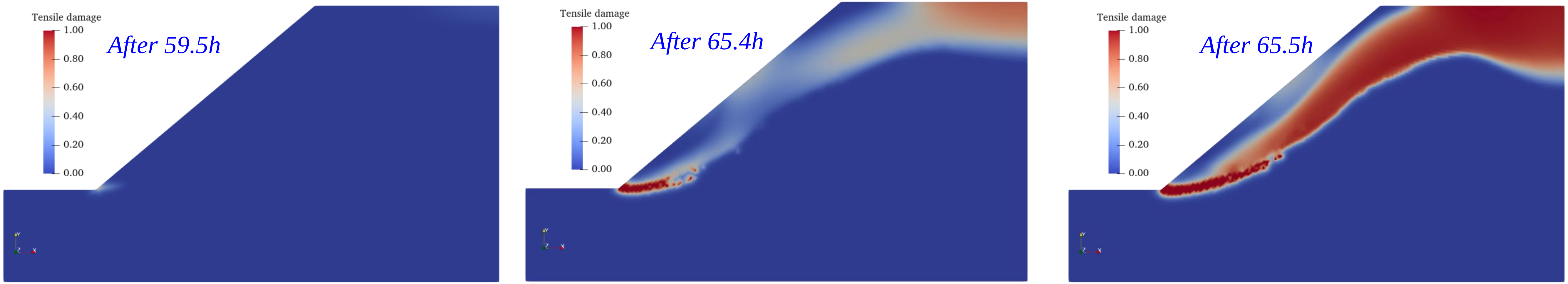



*Figure - Displacement vector*

# Analysis of rainfall induced landslides

*Compressive- shear cracks*




*Tensile cracks*

# Analysis of rainfall induced landslides

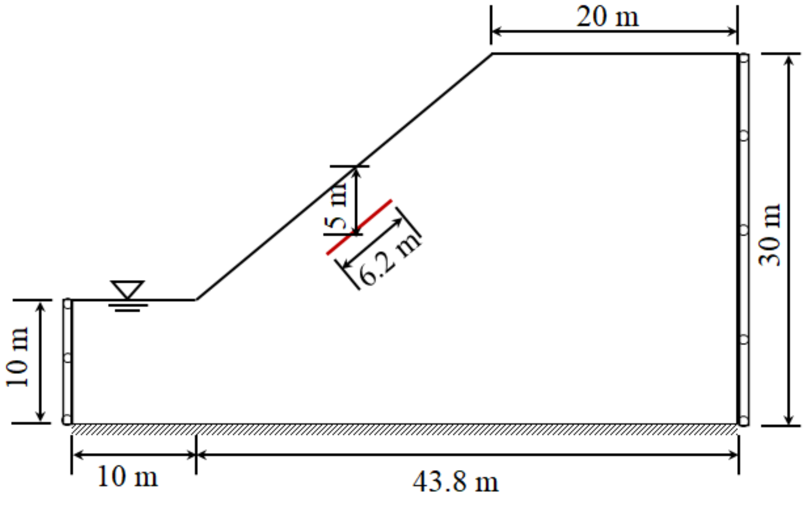



*Figure – Distribution of pre-crack*

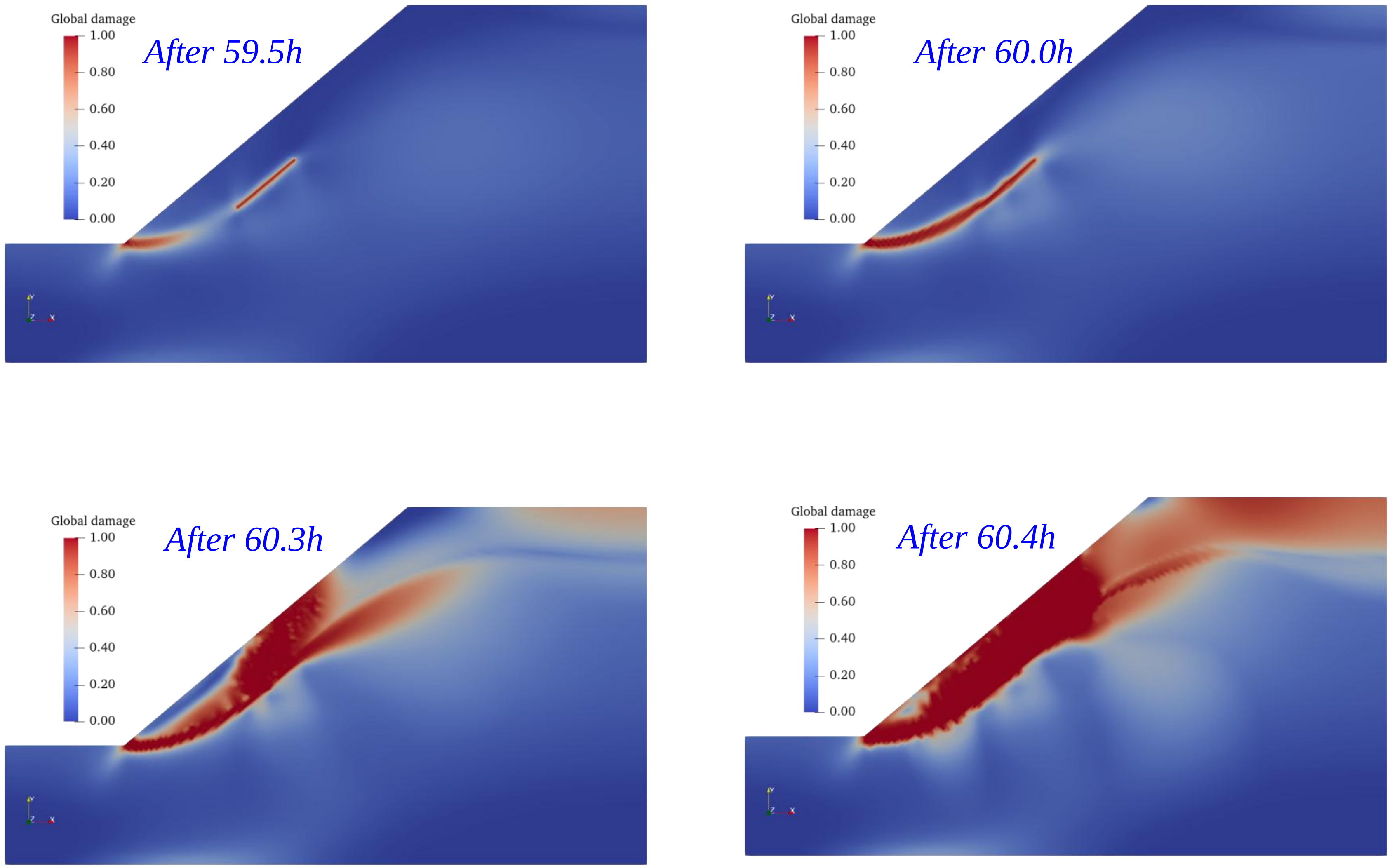


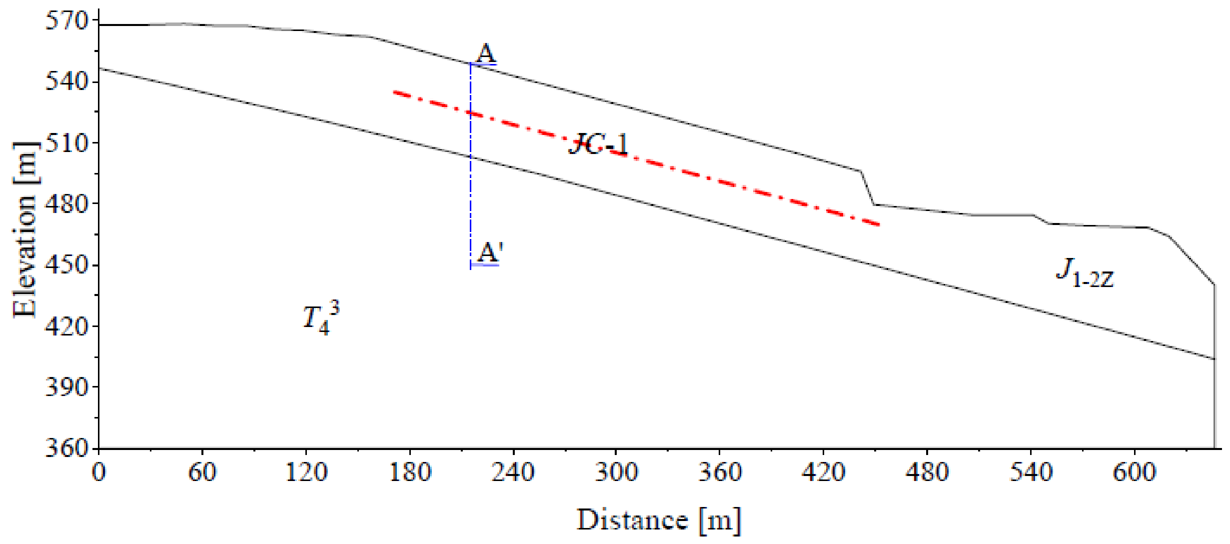

*Figure - Distribution of global damage*

Influences of pre-crack:

- Growth of cracks
- Two-step failure pattern

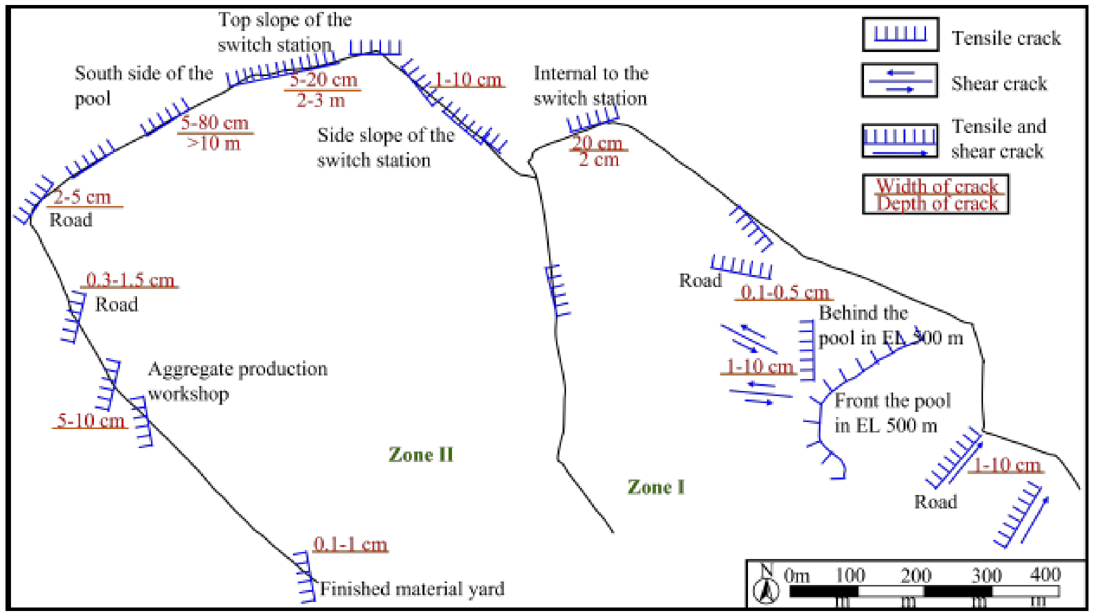
## Analysis of rainfall induced landslides

Description of numerical model:



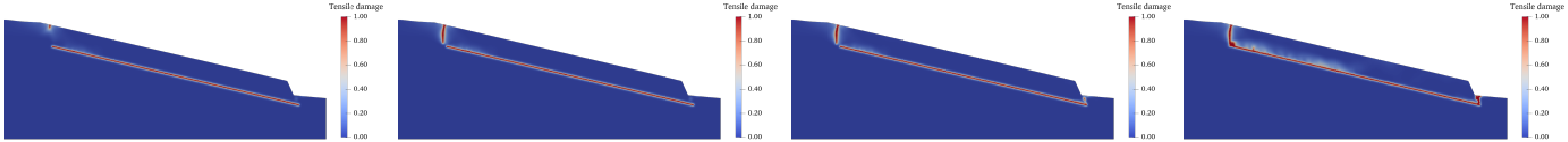

$$\mathcal{H}_0^{t/s}(\boldsymbol{x}) = \begin{cases} \dfrac{g_c^{t/s}}{2l_d}\dfrac{d}{1-d}\left(1-\dfrac{2L(\boldsymbol{x})}{l_d}\right) & L(\boldsymbol{x}) \le \dfrac{l_d}{2} \\ 0 & \text{otherwise} \end{cases}$$

Field investigation of cracks distribution (Zhang et al., 2018):



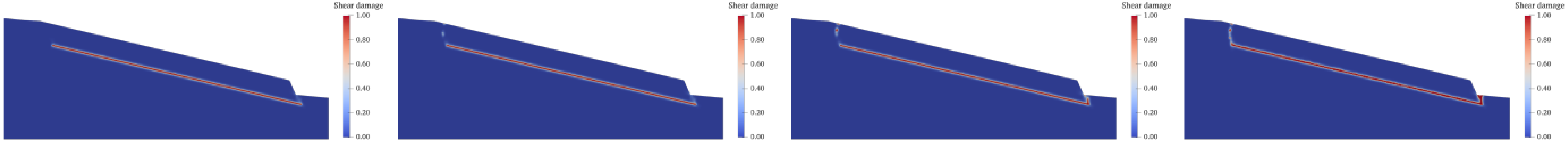
# Analysis of rainfall induced landslides

## Distribution of tensile damage




(a) $RF = 568.9$mm (b) $RF = 598.7$mm (c) $RF = 645.5$mm (d) $RF = 651.3$mm

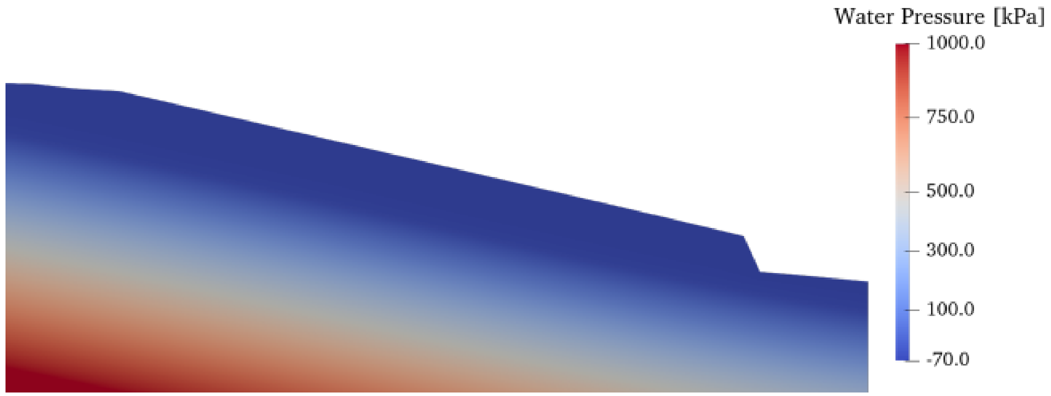
## Distribution of shear damage



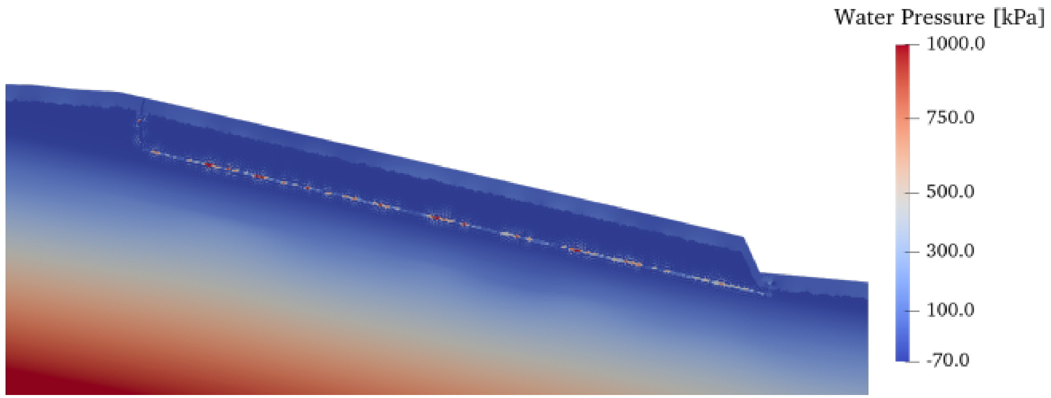

(a) $RF = 568.9$mm (b) $RF = 598.7$mm (c) $RF = 645.5$mm (d) $RF = 651.3$mm

## Analysis of rainfall induced landslides

Distribution of pore water pressure:

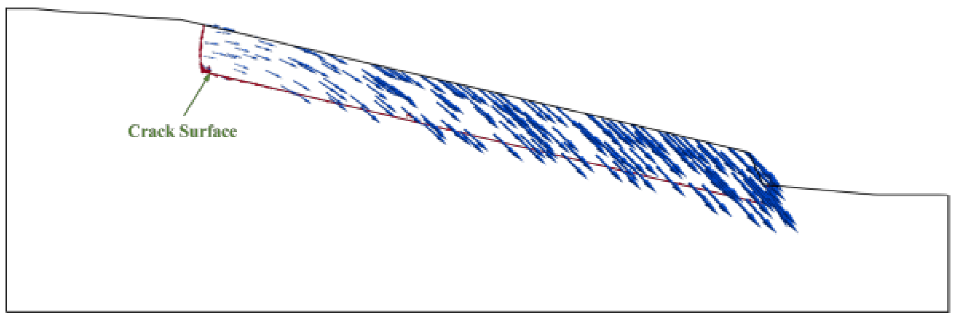



(a) Initial state

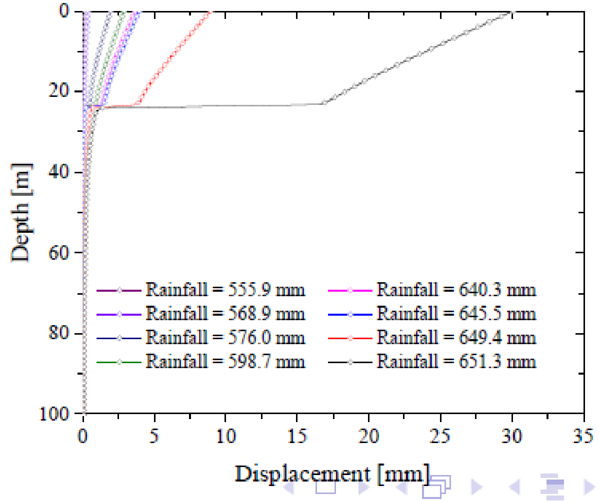



(b) RF = 651.3$mm$

Distribution of displacement:




(a) Displacement vector at RF = 651.3$mm$



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PART FOUR

# CONCLUSIONS AND PERSPECTIVES

## Conclusions

- The proposed method is able to describe the initiation and propagation of localized damage zones and cracks due to rainfall.
- It was found that the shear cracking was the principal failure mechanism of landslides.
- The existence of initial weak zones and fractures enhances the failure process and also affects the cracking pattern

## Perspectives

- Application the proposed numerical method into analysis of reality landslides;
- Considering the material in a slope as a heterogeneous material;
- Proposing a time-dependent phase-field method to simulate the long-term behavior of the slope;
- Taking into account hydrodynamic effects

# Thank you for your attendance !

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